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Structure and Analysis of Planetary Gear Trains

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Abstract
Both compound and internal or external planet gears are considered as well as the paired planet gears. It is stated that the simple planetary gear train consisting of two central gears, one or more planet gears and one arm, has 34 different types. All 34 types can be derived from one general form of the simple planetary gear train. The family tree of planetary gear trains is given. Every kind of joined and united planetary gear trains which have more than two central gears, can be separated into two or more simple types. Uniform mathematical and graphical methods are presented for analysing kinematical, torque and force characteristics of planetary gear trains. Some of the methods are new.

Zusammenfassung—Struktur und Analyse der Planetengetriebe: Z. Léwai
In der Arbeit sind zusammengesetzte innere und äussere Planetengetriebe betrachtet, ebenso wie zweistufige Planetengetriebe. Es wird festgestellt, dass das einfache Planetengetriebe, bestehend aus zwei zentralen Zahnrädern, einem oder mehreren Planetenrädern und einem Arm, 34 verschiedene Arten aufweist. Alle 34 Arten können aus einer allgemeinen Form eines einfachen Planetengetriebes abgeleitet werden. Der Stammbaum der Planetengetriebe ist angegeben. Jede Art von vereinten Planetengetrieben, die mehr als zwei Zentralräder besitzen, können in zwei oder mehrere einfache Arten geteilt werden. Allgemeine methematische und graphische Methoden zur Analyse von kinematischen Drehmoment- und Kraft-Eigenschaften von Planetengetrieben sind angegeben. Einige der Methoden sind neu.

Резюме—Структура и анализ планетарных зубчатых механизмов: З. Левай.
В работе разсмотрены сложные и внешние или внутренние планетарные зубчатые механизмы а также двухступенчатые планетарные механизмы. Устанавливается что простой планетарный механизм, состоящий из двух центральных колес, одного или нескольких сателлитов и одного водила имеет 34 различных типов. Все 34 типа могут быть выведены из одной общей формы планетарнаго механизма. Каждый тип сложнаго и объединеннаго планетарнаго механизма имеющаго больше чем два центральных колеса может быть разделен на два или более простых типов. Общие математические и графические методы даются для анализа кинематических моментов и силовых характеристик планетарных механизмов. Некоторые методы являются новыми.

1. Introduction

A MECHANISM is termed a planetary mechanism if it contains at least one rigid body which is required to rotate about its own axis and at the same time to revolve about another axis (Fig. 1). Points on this body will generate epicycloids or hypocycloids. Therefore a planetary mechanism is often called an epicyclic or cyclic mechanism.

A planetary mechanism can be obtained by mounting a rigid body, often referred to as a planet, on a crank pin. Theoretically the crank and the planet can be driven by different

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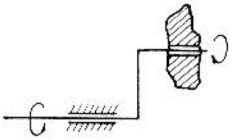


Figure 1.

motors (Fig. 2). It is not necessary to mount the motor, driving the planet, on the crank pin itself (Fig. 3). In practice, the planet is rotated by rolling it either on the outside or on the inside of a stationary gear (Figs. 4 and 5). The axis of the stationary gear must be collinear with the axis of the crank. The stationary gear (sun gear or ring gear) can be referred to as the central gear. The crank is generally called the arm or carrier.

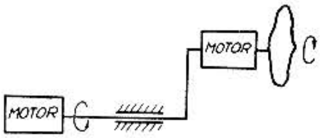


Figure 2.

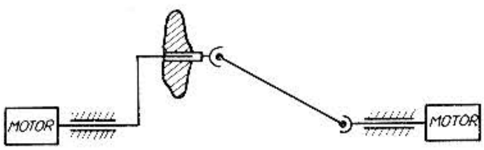
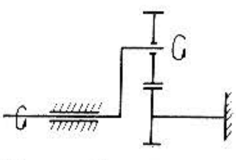


Figure 3.



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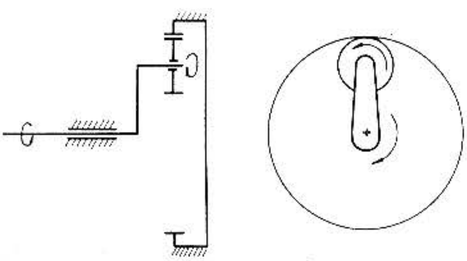
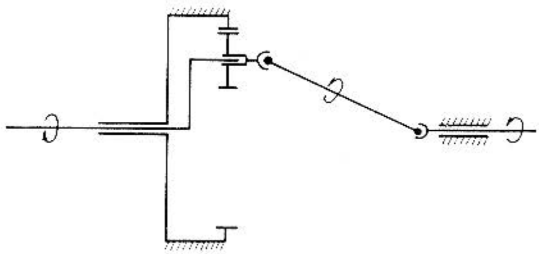


Figure 5.

A mechanism which consists of one central gear, one or more planet gears and one arm, carrying the planet gears, can be called an elementary planetary gear train (elementary P.G.T.).

In case the motion of the planet gear is required directly, the shaft of the planet gear can, for example, be coupled to another shaft (output shaft) by universal joints (Fig. 6). However, the rotation of the planet gear is seldom used directly. Generally, a second central gear is driven by the planet gear (Fig. 7). The fact that more than one (usually three) planet gears are placed between the two central gears does not change the character of the P.G.T. (Fig. 8). In this case the planet gears can be referred to as parallel gears because each planet gear is in mesh with both of the central gears.



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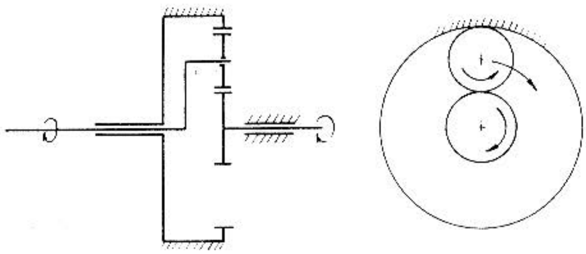

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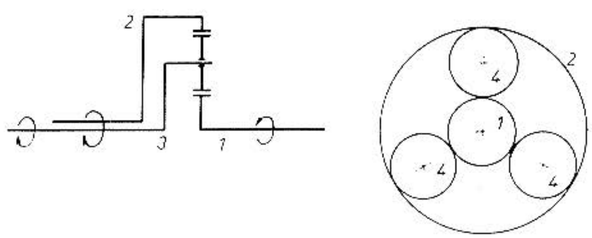


Figure 8.

A mechanism which consists of two central gears, one or more planet gears and one arm, carrying the planet gears, can be called a simple planetary gear train (simple P.G.T.).

It is noted that in Fig. 8, there is no fixed central gear; all of the gears can rotate as well as the arm. This is the general case; a fixed gear or fixed arm is a special case.

In Fig. 8, some of the symbols used are shown; the numbers 1 and 2 always indicate the central gears, 3 the arm and 4 the planet gear.

Often the same planet meshes with two central gears on different pitch circles or with gears being in two different planes (Fig. 9). This type is known as a compound planet gear.

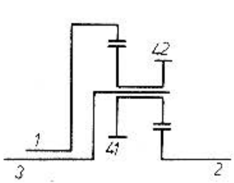


Figure 9.

Planet gears can be placed not only in parallel with each other but also in series, that is one after the other (paired planet gears) (Fig. 10). In this case, there is no planet gear mesh in directly with both central gears; each planet gear meshes with only one central gear and another planet gear. Application of paired planet gears cause a change in direction of rotation that is in the character of the P.G.T. A set of three planet gears in series is meaningless because the direction of rotation will be the same again as in the case of single planet gear.

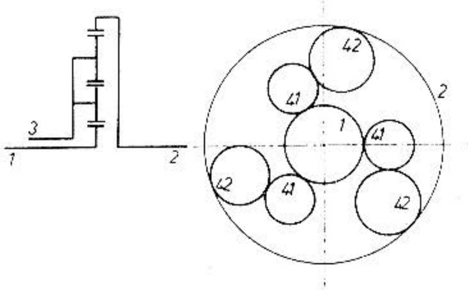


Figure 10.

2. Conception of P.G.T. Types

In conclusion it can be said that a simple P.G.T. is the most advanced one if it is made with paired compound planet gears (Fig. 11). In this figure, the dotted lines signify that both central gears can be either an external or an internal gear. The same P.G.T. is illustrated in Fig. 12, where the axes of the planet gears are not parallel with that of the central gears. It must be noticed that application of bevel gears does not change the character of a P.G.T., it can only modify the numerical values of the characteristics (Fig. 13). Therefore bevel gears will not be dealt with here.

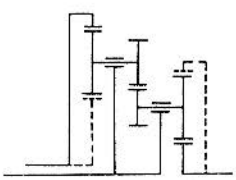


Figure 11.

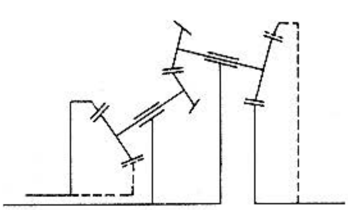


Figure 12.

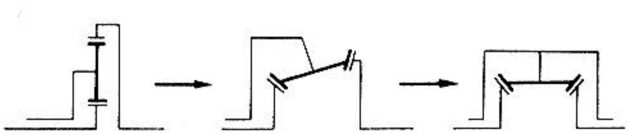


Figure 13.

Notice that the differences in character only result in new P.G.T. types. The character depends on

(1) how many gears there are in the train; and

(2) which of the gears are internal or external.

The character also remains unchanged if the central gears or their shafts or the arm are set in different sequence or configuration. One does not get a new P.G.T. type by reflection, either (Fig. 14). The circumstance as to which shaft is stationary or input or output has also no effect on the question of type.

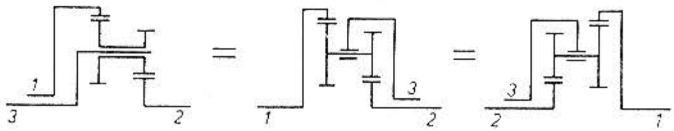
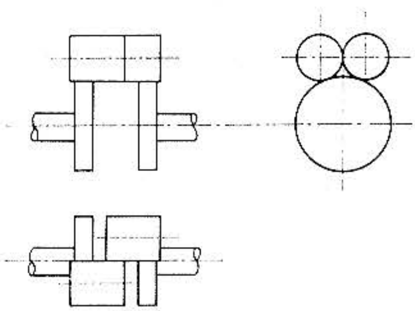


Figure 14.

There is a possibility of naming P.G.T. types by letters. If the letter *P* stands for external gears (both central and planet gears) and the letter *N* for internal gears; and if the letter, whether *P* or *N*, when it refers to a planet gear, is put in brackets, then one can write for the P.G.T.'s given in Fig. 14: Type *N(PP)P* or its reflection: Type *P(PP)N* which, naturally, is the same as Type *N(PP)P*. Further examples: in Fig. 10, Type *P(P)(P)N*, and in Fig. 8, Type *P(P)N* are shown.

Applying paired planet gears, the axes of the central gears and that of the pair of planet gears generally do not lie in the same plane. In other words, their centers do not lie along one straight line. In some cases the centers of both of the paired planet gears are at the same distance from the center of the sungears (Fig. 15). Which of the paired planet gears will be closer to the axis of the central gears depends on the sizes and not on the character of the P.G.T. Therefore, it is not necessary (and sometimes impossible) to show the actual sizes or distances in a symbolical representation (Fig. 16).



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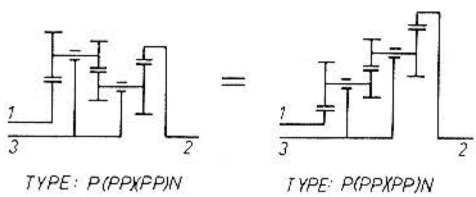

Figure 16.

It was mentioned above that central gears can be geared either internally or externally. Accordingly, one can get different P.G.T. types with uniform planet gears (Fig. 17).

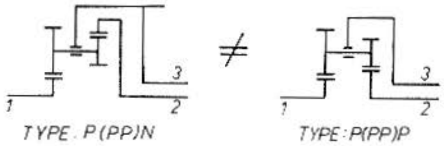

Figure 17.

The exchange of an external gear for an internal gear can be taken as a decrease in the diameter from some positive value to some negative one (Fig. 18). In naming the P.G.T.

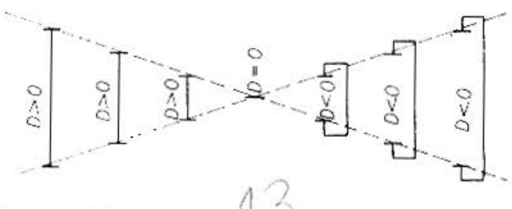

Figure 18. 13

types by letters, the letter P means positive diameter and the letter N means negative diameter. A decrease in the diameter or change of its sign can be done to planet gears, too (Fig. 19), resulting in new P.G.T. types again. Of course, the absolute values of diameters are limited. Sketches b and c in Fig. 19, for example, cannot be realized.

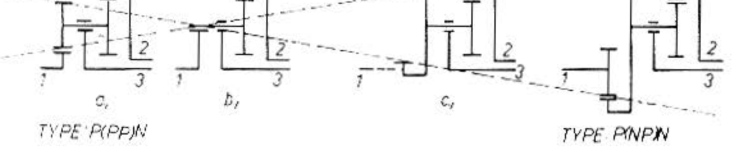

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Figure 19.

3. Derivation of P.G.T. Types

It is now advantageous to determine how many types of simple P.G.T. exist. The derivation of types from the general form given in Fig. 11, is a process of changing diameters. In certain cases a change in diameters without exchange of sign can also give new types of P.G.T. This occurs when the diameter of certain gears become equal to each other and, consequently, the number of gears decreases (Figs. 20 and 21). An exchange of sign will result in new types in most cases.

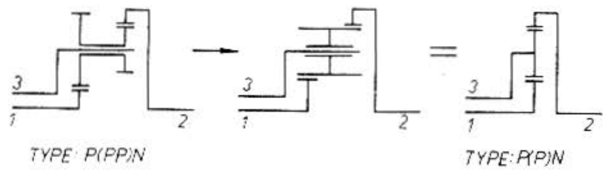

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Figure 20.

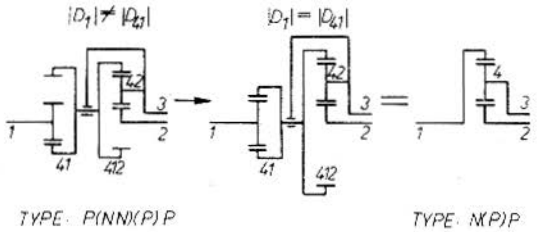

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Figure 21.

The general form is given again in Fig. 22, indicating the necessary index numbers.

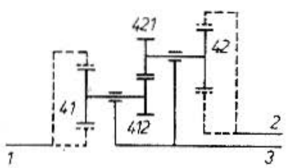

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Figure 22.

Table 1 contains all possible variations of external and internal gears. The variants which can practically be realized are denoted by bold-faced letters and given serial numbers. Some of the variants are reflection of each other. Since a reflection does not result in a new P.G.T. type, the type numbers which are being repeated are denoted by thin figures. Thin letters and the sign "–" mean that the variation cannot be realized practically. For instance, two internal gears cannot be in mesh with each other.

On the basis of Table 1, it can be said that the simple P.G.T. has 34 different types.

Figure 23 illustrates the complete family tree of the simple P.G.T. It can be seen which special type is derived from which more general type. In this family tree are also indicated either the signs or the proportions of the various diameters as related to the variants. The heavier arrows show the types made by changing the sign of diameters, thin arrows show the types made by reducing the number of the planet gears. The types in the circles are those most generally used.

At first it may happen that a P.G.T. does not appear to correspond to any one of the 34 types. There can be two reasons for this: the order and configuration of the gears and shafts may be unusual or the P.G.T. in question is not a simple one but made up of joined or united P.G.T.-s.

Figure 24 illustrates examples for unusual configuration. The first P.G.T. corresponds to type No. 8, the second one to type No. 10.

Table 1

No.	Type	No.	Type	No.	Type
1	**P(P)N**	—	P(P)P	—	N(P)N
—	P(N)N	—	P(N)P	—	N(P)N
2	**P(PP)N**	**16**	**P(PP)P**	**31**	**N(PP)N**
3	**P(NP)N**	**17**	**P(NP)P**	—	N(NP)N
—	P(PN)N	**17**	**P(PN)P**	—	N(PN)N
—	P(NN)N	**18**	**P(NN)P**	—	N(NN)N
4	**P(P)(P)N**	—	P(P)(P)P	—	N(P)(P)N
—	P(N)(P)N	—	P(N)(P)P	—	N(N)(P)N
—	P(P)(N)N	—	P(P)(N)P	—	N(P)(N)N
—	P(N)(N)N	—	P(N)(N)P	—	N(N)(N)N
5	**P(P)(PP)N**	**19**	**P(P)(PP)P**	**32**	**N(P)(PP)N**
6	**P(N)(PP)N**	**20**	**P(N)(PP)P**	—	N(N)(PP)N
7	**P(P)(NP)N**	**21**	**P(P)(NP)P**	—	N(P)(NP)N
—	P(P)(PN)N	**22**	**P(P)(PN)P**	—	N(P)(PN)N
—	P(N)(NP)N	—	P(N)(NP)P	—	N(N)(NP)N
—	P(N)(PN)N	—	P(N)(PN)P	—	N(N)(PN)N
—	P(P)(NN)N	**23**	**P(P)(NN)P**	—	N(P)(NN)N
—	P(N)(NN)N	—	P(N)(NN)P	—	N(N)(NN)N
8	**P(PP)(P)N**	**19**	**P(PP)(P)P**	**32**	**N(PP)(P)N**
9	**P(NP)(P)N**	**22**	**P(NP)(P)P**	—	N(NP)(P)N
—	P(PN)(P)N	**21**	**P(PN)(P)P**	—	N(PN)(P)N
—	P(PP)(N)N	**20**	**P(PP)(N)P**	—	N(PP)(N)N
—	P(NN)(P)N	**23**	**P(NN)(P)P**	—	N(NN)(P)N
—	P(NP)(N)N	—	P(NP)(N)P	—	N(NP)(N)N
—	P(PN)(N)N	—	P(PN)(N)P	—	N(PN)(N)N
—	P(NN)(N)N	—	P(NN)(N)P	—	N(NN)(N)N
10	**P(PP)(PP)N**	**24**	**P(PP)(PP)P**	**33**	**N(PP)(PP)N**
11	**P(NP)(PP)N**	**25**	**P(NP)(PP)P**	—	N(NP)(PP)N
12	**P(PN)(PP)N**	**26**	**P(PN)(PP)P**	**34**	**N(PN)(PP)N**
13	**P(PP)(NP)N**	**26**	**P(PP)(NP)P**	**34**	**N(PP)(NP)N**
—	P(PP)(PN)N	**25**	**P(PP)(PN)P**	—	N(PP)(PN)N
14	**P(NN)(PP)N**	**27**	**P(NN)(PP)P**	—	N(NN)(PP)N
15	**P(NP)(NP)N**	**28**	**P(NP)(NP)P**	—	N(NP)(NP)N
—	P(NP)(PN)N	**29**	**P(NP)(PN)P**	—	N(NP)(PN)N
—	P(PN)(NP)N	—	P(PN)(NP)P	—	N(PN)(NP)N
—	P(PN)(PN)N	**28**	**P(PN)(PN)P**	—	N(PN)(PN)N
—	P(PP)(NN)N	**27**	**P(PP)(NN)P**	—	N(PP)(NN)N
—	P(NN)(NP)N	—	P(NN)(NP)P	—	N(NN)(NP)N
—	P(NN)(PN)N	**30**	**P(NN)(PN)P**	—	N(NN)(PN)N
—	P(NP)(NN)N	**30**	**P(NP)(NN)P**	—	N(NP)(NN)N
—	P(PN)(NN)N	—	P(PN)(NN)P	—	N(PN)(NN)N
—	P(NN)(NN)N	—	P(NN)(NN)P	—	N(NN)(NN)N

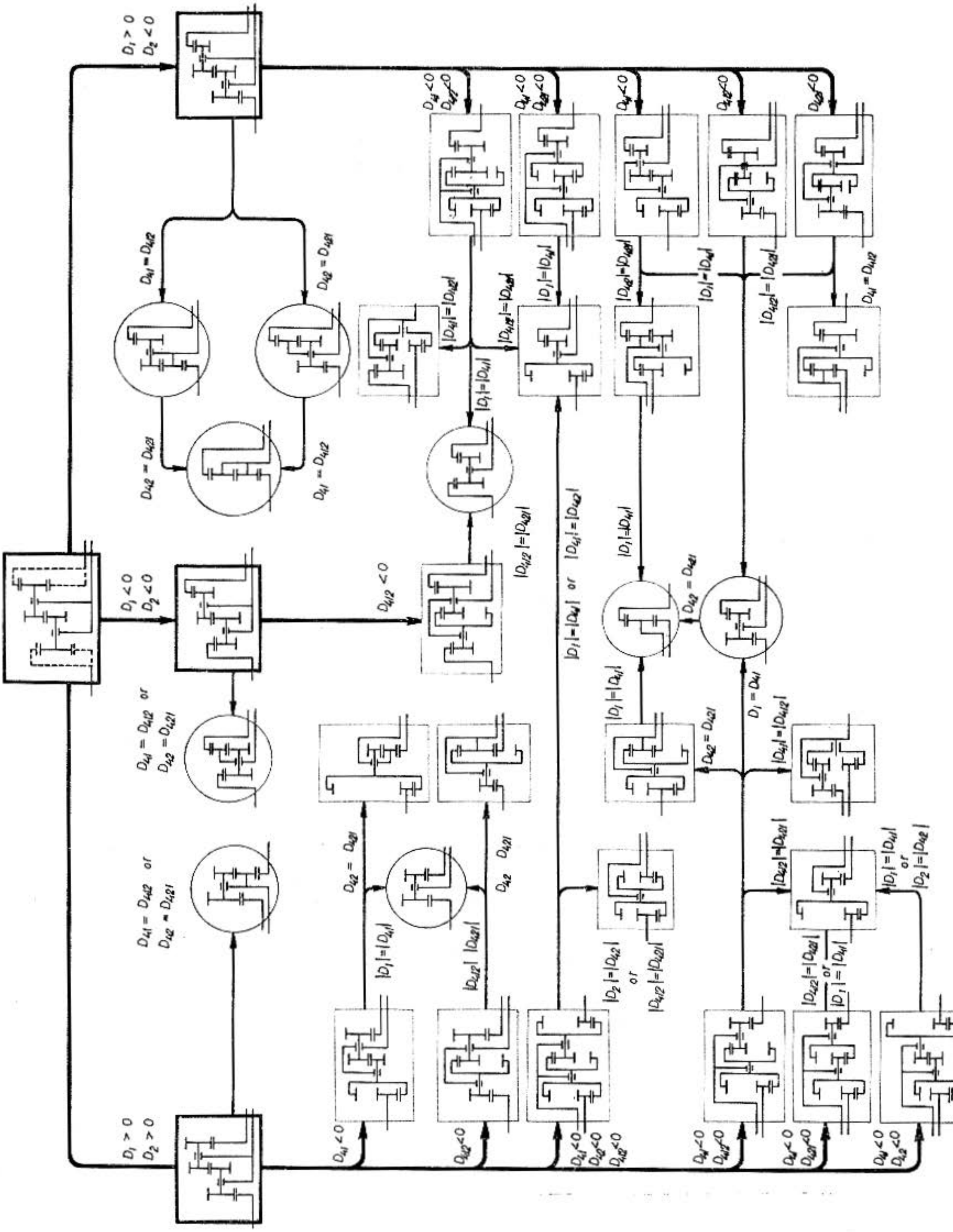


Figure 23.

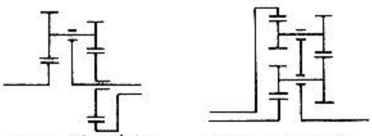


Figure 24.

A joined P.G.T. can be recognized by the fact, that in every case, it has more than two central gears. In every case, joined P.G.T.-s can be separated into two or more simple P.G.T.-s. The joined P.G.T.-s shown in Fig. 25 can be separated into type No. 16 and type No. 1.

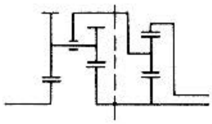


Figure 25.

The separation is not so obvious if one of the planet gears of the first simple P.G.T. is united with one of the planet gears of the second simple P.G.T. In this case the related central gears are united as well. This may occur when the sizes of these planet gears and their arm radii are equal to each other. Figure 26 shows the same type of joined P.G.T.-s

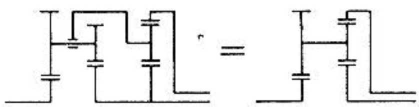


Figure 26.

given in Fig. 25, but with two of the planet gears united. Joined P.G.T.-s with united gears can be called united P.G.T.-s. Figures 27 and 28 also illustrate united P.G.T.-s. The united P.G.T.-s in Figs. 29 and 30 have been constructed of three simple P.G.T.-s.

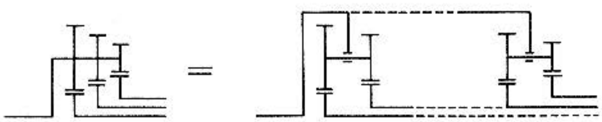


Figure 27.

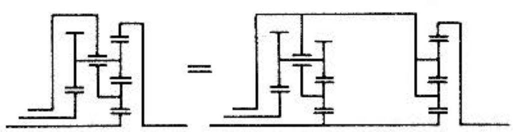
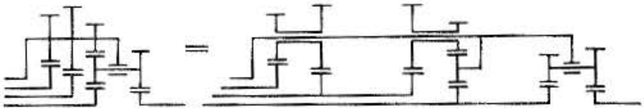
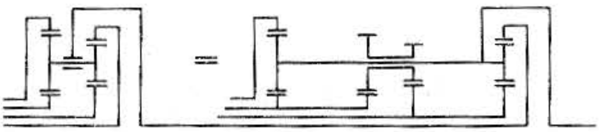


Figure 28.



TYPE 2×P(PP)P+P(PP)(P)P = TYPE P(PP)P + TYPE P(PP)(P)P + TYPE P(PP)P

Figure 29.



TYPE 2×P(P)N+P(PP)P = TYPE P(P)N + TYPE P(PP)P + TYPE P(P)N

Figure 30.

4. Analytical Methods of P.G.T.'s Analysis

There are analytical and graphical methods for the theoretical investigation and design of a P.G.T. Some of these methods have been known previously.

According to the Willis's analytical method [38] the simple P.G.T. can be characterized by the "basic ratio" which is the ratio of the angular velocities of the two central gears relative to the arm. Choosing symbol b for the basic ratio, it can be written as

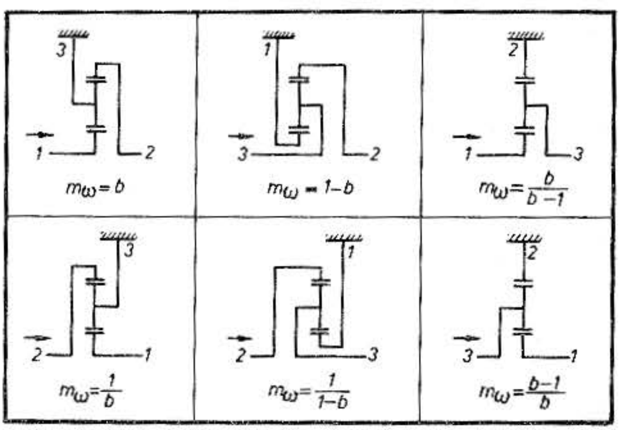
$$b=\frac{\omega_2'}{\omega_1'}, \tag{1}$$

where $\omega_1'=\omega_1-\omega_3$ and $\omega_2'=\omega_2-\omega_3$; therefore

$$b=\frac{\omega_2-\omega_3}{\omega_1-\omega_3}. \tag{2}$$

The reason for using the symbol b for the basic ratio of a P.G.T. was that it is different from the known velocity ratio m_ω or train value e. The basic ratio b of a given P.G.T. is fixed and it characterizes the P.G.T. uniquely. The velocity ratio or train value, at the same time, can be different for a given P.G.T., depending on how the P.G.T. is applied (Table 2).

Table 2.



Equation (2) can be written as follows:

$$b\omega_1-\omega_2-(b-1)\omega_3=0. \tag{3}$$

This is the basic kinematical equation of the simple P.G.T. From it one can determine any one of the three angular velocities when the other two of them are known together with the value of b. One could also calculate from equation (3) the angular velocity ratio between any two shafts when the angular velocity of the third shaft and b are given.

The basic ratio b of a given P.G.T. can be determined from the diameters of the gears; namely, assuming the arm to be stationary the basic ratio will be identical with the angular velocity ratio and will be inversely proportional to the diameters of the gears with negative sign. The formula for the general form of the simple P.G.T. is as follows:

$$b=-\frac{D_1\times D_{42}\times D_{412}}{D_2\times D_{41}\times D_{421}}. \tag{4}$$

The sign of the diameters must be taken into account; accordingly, b may have either a positive or a negative value. The formula (4) becomes shorter for simpler types. For example, the types Nos. 3, 16, 31 are the result of the condition:

$$\frac{D_{421}}{D_{412}}=-1,$$

thus their formula is:

$$b=\frac{D_1\times D_{42}}{D_2\times D_{41}}. \tag{5}$$

For the type No. 1, also $D_{41}=D_{42}$, therefore

$$b=\frac{D_1}{D_2}, \tag{6}$$

but for the type No. 4 ($D_{41}=D_{412}$ and $D_{42}=D_{421}$):

$$b=-\frac{D_1}{D_2} \tag{7}$$

Since the simple P.G.T. has three main elements (two central gears and one arm or rather their shafts), the torques acting on them are always in equilibrium (uniform rotation assumed), thus:

$$T_1+T_2+T_3=0. \tag{8}$$

Furthermore, when $\omega_3=0$, one can write (by applying equation 3):

$$\frac{T_2}{T_1}=-\frac{\omega_1}{\omega_2}=-\frac{1}{b}. \tag{9}$$

This gives

$$T_2=-\frac{1}{b}T_1. \tag{10}$$

From equations (8) and (10) follows that

$$T_3 = -T_1 - T_2 = \left(\frac{1}{b} - 1\right) T_1. \tag{11}$$

Therefore equations (10) and (11) can be written in the form:

$$T_1 : T_2 : T_3 = 1 : \left(-\frac{1}{b}\right) : \left(\frac{1}{b} - 1\right). \tag{12}$$

This equation gives the basic proportions of the torques for the ideal simple P.G.T.

Dividing each term in equation (12) by the appropriate radii, gives the basic proportions of the tangential forces for the ideal simple P.G.T.:

$$F_1 : F_2 : F_3 = \frac{1}{r_1} : \left(-\frac{1}{r_2 b}\right) : \frac{1}{r_3}\left(\frac{1}{b} - 1\right). \tag{13}$$

Multiplying each term in equation (12) by the appropriate angular velocities, gives the basic proportions of the power for the ideal simple P.G.T.:

$$P_1 : P_2 : P_3 = \omega_1 : \left(-\frac{1}{b}\right)\omega_2 : \left(\frac{1}{b} - 1\right)\omega_3. \tag{14}$$

5. Graphical Methods of P.G.T. Analysis

Kutzbach used a graphical method for the kinematical analysis of the P.G.T. [14]. This method is based on the principle of kinematics that the instantaneous velocity of any point on a rigid body being in plane motion can be determined if the velocities of two other points on it are already known. A planet gear has three points which are of interest: its center, which is coincident with the center of arm pin, and its two pitch points, which are coincident with the pitch points of the central gears. Consequently, if the instantaneous velocities of these three points on the planet gear are known then the instantaneous velocity of one point on each P.G.T. element is also known. The tangential velocities are represented by vectors (Fig. 31). The endpoints of the vectors must lie in a straight line. If two vectors are given, the third one can be drawn easily. By projection of the vectors on some common radius one obtains straight lines, the lengths of which are proportional to the angular velocities of the P.G.T. shafts.

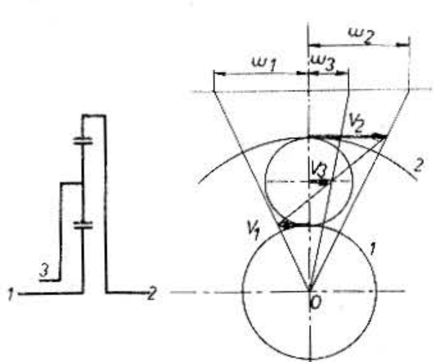


Figure 31.

Graphical methods for the analysis of the tangential forces and torques have been proposed by the author [19]. The method of drawing a force diagram is shown in Fig. 32. By taking first the force F_3 applied on the arm pin, one has to draw three lines in the directions shown by the arrows to get the force F_2 (sketch *a*). One can get the force F_1 in the same way (sketch *b*). The sketch *c* shows all three forces together.

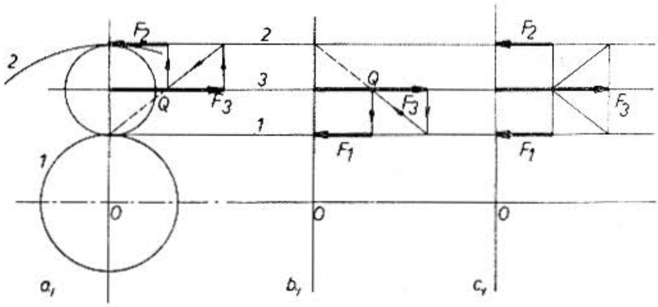


Figure 32.

The torque diagram looks like the force diagram with the only difference that at the intersection Q, the third line must be dropped in the radial direction instead of vertically (Fig. 33).

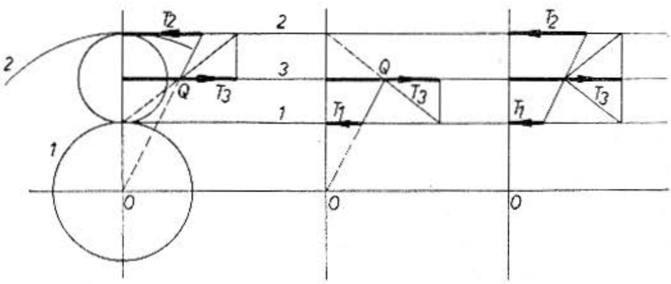


Figure 33.

Notice that the sense of the force vectors or torque lines must be specified as follows: the inside vector must have a sense opposite to that of the two outside vectors regardless of which P.G.T. element it belongs to, provided that all three vectors are on the same side of the vertical central line. If one of the outside vectors is on the opposite side of the vertical central line then its sense must be changed.

The above graphical methods can be used for compound planet gears as well as for internal planet gears (Figs. 34 and 35). Note the sense of T_1 in Fig. 35.

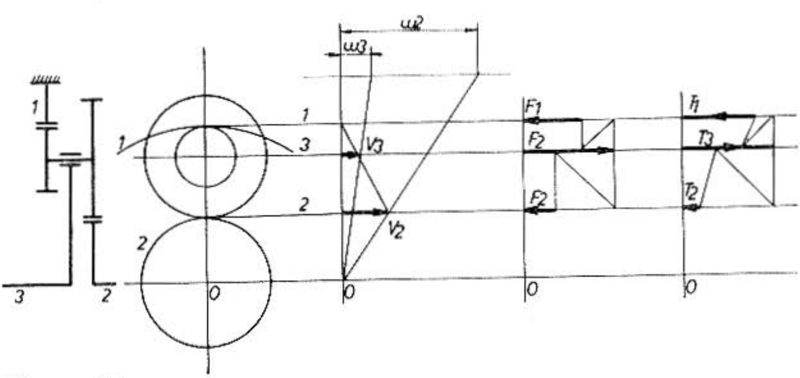


Figure 34.

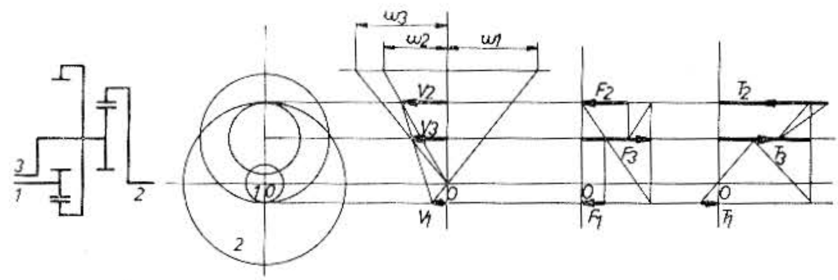


Figure 35.

However, when paired planet gears are used in a P.G.T. then the methods must be supplemented. In this case, there is no planet gear having coincident points with both of the central gears. For P.G.T. with paired planet gears, the author has proposed the following method (the proof of this method is given in [16] and [18]) (Fig. 36).

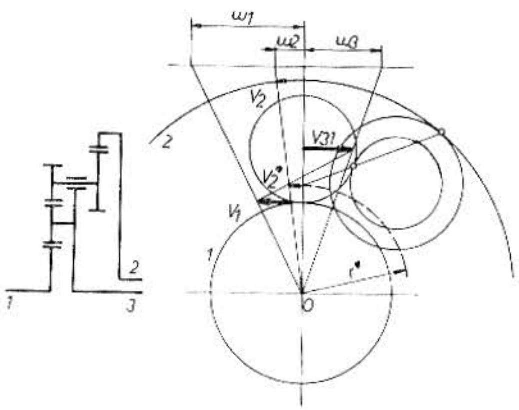


Figure 36.

Select one of the paired planet gears (for example, the gear 31), as the main one. Erect a line through the centers of the main planet gear and the central gears. The butt of vector V_{31} is at the center of the main planet gear. The butt of vector V_1 is at the pitch point of the main planet gear and the central gear. The problem is to find the location of the third vector. To accomplish this drop a line through the pitch points of the other planet gear which is now an auxiliary one. The intersection of this line with the vertical is the butt of the third vector. Once this butt is found, one can sketch all the vectors and determine the angular velocities in the known way.

It must be noted that although the third vector V_2^* belongs to the central gear 2, it gives the tangential velocity on the reduced radius r_2^* and not on its pitch circle. If one is interested in the tangential velocity V_2, one can find it by simple projection.

Having found the reduced radius r^*, one can also construct the force and torque diagrams by the known methods. In Fig. 37, the velocity diagram of the previous P.G.T. is shown again but in this case, the other planet gear had been chosen as main planet gear. The results, naturally, are the same. Notice that force F_1^* is different from force F_1, the latter being the force acting on the pitch point of the central gear 1. The force F_1 can be obtained by reduction of F_1^* from radius r_1^* to radius r_1 (see dotted lines in sketch c). On the other hand, the force F_2 is the actual force acting on the pitch point of the central gear 2. Force F_3 is the actual tangential component of the force acting on the arm pin 32, provided

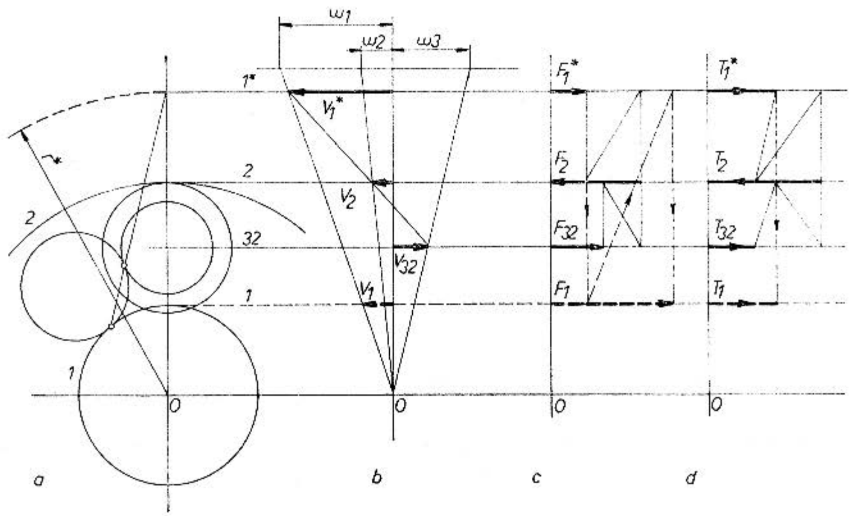


Figure 37.

that the force acting on arm pin 31 is imagined to be reduced to arm pin 32. As it is known certain K_1 and K_2 forces appear on both arm pins. They can be drawn from F_1 or F_2 (Fig. 38). Force F_{31} or F_{32} corresponds to the tangential component of the resultant of K_1 and K_2, reduced to radius r_{31} or r_{32}.

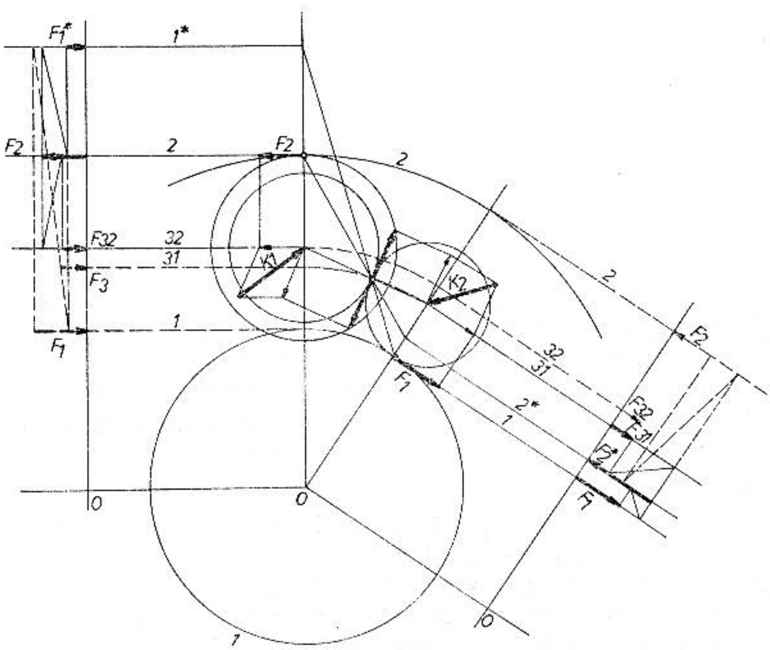


Figure 38.

Torque T_1^* corresponds to T_1 immediately (Fig. 37).

Two further examples are illustrated in Figs. 39 and 40. One can find internal planet gears in Fig. 40.

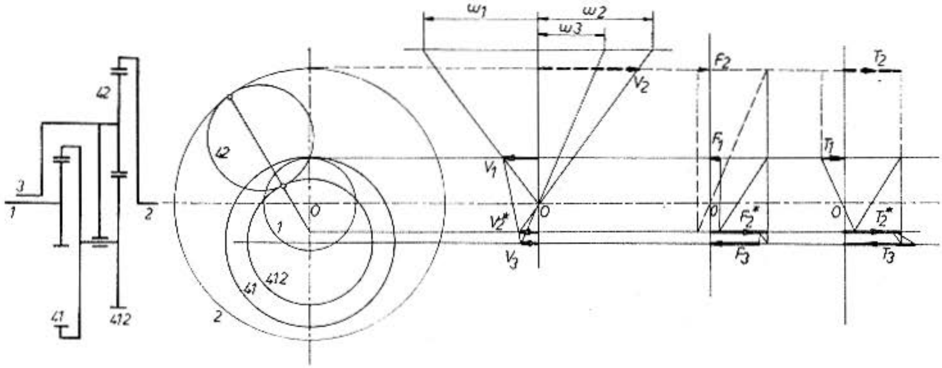


Figure 39.

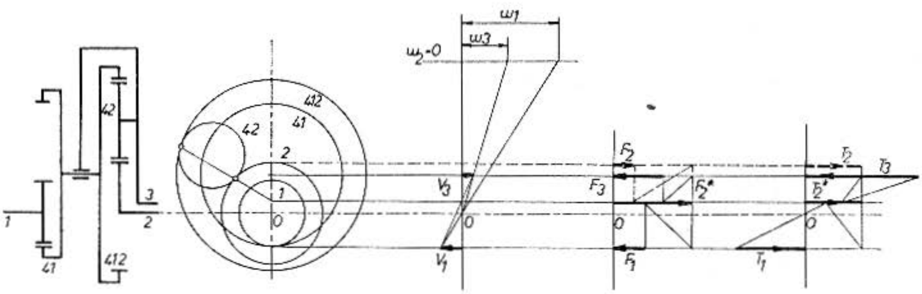


Figure 40.

Both mathematical and graphical methods can be used for joined P.G.T.-s as well as for united P.G.T.-s once the constituent parts have been recognized and separated.

Finally, it must be mentioned that the reduced radius r^* has its own significance. Radius r^* is the radius of a central gear of a certain P.G.T. without auxiliary planet gears but with the same characteristics as the original P.G.T. which has paired planet gears. Since one can choose any one of the paired gears as a main one, one can get two different reduced radii r^* (r_1^* and r_2^*). Thus every simple P.G.T. with paired planet gears corresponds to two simple P.G.T. without auxiliary planet gears. That is, one can be replaced by the other (Fig. 41).

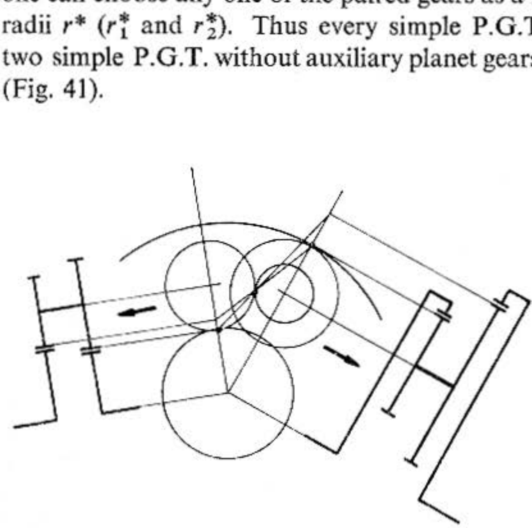


Figure 41.

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