TECHNICAL UNIVERSITY OF BUILDING, CIVIL AND TRANSPORT ENGINEERING
PROFESSORATE FOR MOTOR VEHICLES

NUMBER 29

dr. ZOLTÁN LÉVAI

THEORY OF EPICYCLIC GEARS AND EPICYCLIC CHANGE-SPEED GEARS

Budapest
1966

Felelős kiadó:
Dr. LÉVAI ZOLTÁN

66661 MTA KESZ Sokszorosító. F. v.: Szabó Gyula

CONTENTS

Page

INTRODUCTION

The literature of the epicyclic gear as regards its character may be classified in several groups.

We find parts refering to the theory of epicyclic gears in works of relatively numerous authors, the majority of them, however, sticks to the repetition of one and the same elementary connection (1, 5, 6,,7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 21, 22, 23, 24, 27, 34, 42, 58, 62, 84, 85, 86, 87, 88, etc.). This question is summarized the best by Terplán (83) who traces his research work back to the elementary epicyclic gear.

Another group of the literature of epicyclic gears deals with the problems of designing, strength dimensioning, manufacture of the simple epicyclic gear (19, 25, 28, 32, 40, 54, 55, 59, 60, 64, etc.).

The literature of the change-speed gear with epicyclic gear as opposed to that of the epicyclic gear itself, is exceedingly one-sided. We can find theoretical examinations only in connection with the individual change-speed stages (29, 30, 34, 35, 38, 43) respectively with the hydromechanical stages (37, 39, 41, 44, 45, 56, 67, 95). The multistage change-speed gear as a complex structure remained outside of the circle of interest of the researchers; some material related to the synthesis of the change-speed gear with epicyclic gear can be found only at Krjukov (35). While there is no theory and mainly no analysis of the change-speed gear, for the realized change-speed gear we find a rather wide literature with a descriptive character in the first place. Here we refer only to some of the great number of publications describing the different products (47, 48, 49, 50, 51, 52, 53). It has to be mentioned separately that a great number of patents refer to change-speed gears with epicyclic gears; of these also only a few will be called up (68-81, 89-93).

In course of studying the literature it was stated by us that there is the greatest lack of literature regarding the theoretical relations of the change--speed gear with epicyclic gear. We did not find such unified system or method which could render possibility for the comprehensive analitical examination of the epicyclic structures having possibilities of innumerable variations and combinations. Notwithstanding the fact, that the laws of the epicyclic gear structures have already been examined manysidedly, we have no such coherent theory which would unite in a comprehensive system the epicyclic gears and the epicyclic gear structures, resp. would show organic connections between the laws of the individual concrete forms of realization, moreover, would give possibility to find the forms of realization of the system unknown up till now.

We ventured to make up this lack in this paper. Characteristic of our method was the fact that all the results of the reasearches made up till now were dealt with according to a uniform viewpoint and we endeavoured to fill up the leakages.

As our examinations referred strictly to the theory of the epicyclic gear and the change-speed gear with epicyclic gear, everywhere the ideal, loss-free epicyclic gear was taken for base, therefore the examination of the epicyclic gear efficiency, as a matter of course, was left out of the analysis.

I. THE ELEMENTARY, THE SIMPLE AND THE COMPLEX EPICYCLIC GEARS

1. THE ELEMENTARY EPICYCLIC GEAR

Basic characteristic of the epicyclic gear or the epicyclic driving gear is that it contains at least one such geared-, frictional-, etc. wheel which is not only turning around its own shaft resp. is able to do so but revolves at the same time around an other shaft or is able to do so i.e. has such a criculating wheel the points of which travel along an epicycle. The epicyclic wheel is made to perform this epicyclic motion by rolling it down on the periphery of another wheel (Fig. 1.).

Accordingly, the socalled elementary epicyclic gear consists of two wheels: of a socalled central wheel (1) and an epicyclic wheel (4). A separate structure, the socalled crank arm (3) serves for fitting the epicyclic wheel with bearings resp. for its guiding. The parts of the epicyclic gear are so the central wheel, the epicyclic wheel and the crank arm.

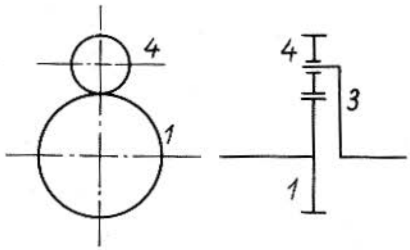


Fig. 1.

2. THE SIMPLE EPICYCLIC GEAR

21. Classification of the simple epicyclic gear

Since direct transmission of the epicyclic wheel's motion meets with difficulties, the elementary epicyclic gear is not much used in practice. Practical application of the epicyclic gear will be essentially simplified by transmitting the epicyclic wheel's motion by an other central wheel. We get in this way to the socalled simple epicyclic gear consisting already of four parts: of two central wheels, a crank arm and an epicyclic wheel (Fig. 2.). I.e. by the simple epicyclic gear the task of the epicyclic wheel mounted on the crank arm may be determined also in this way: ensuring connection between the two central wheels, that is, the direct utilization of the rotating motion of the epicyclic wheel will be discarded with. This means that the shaft of the epicyclic wheel embedded in bearing in the crank arm will not be led out. It is characteristic, therefore, of the simple epicyclic gear that is has three such parts the rotational motion of

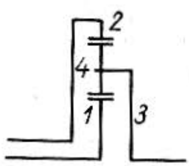


Fig. 2.

which may be utilized directly; all three parts fulfill the condition of the uniaxiality and these three parts are the two central wheels and the crank arm. The epicyclic wheel itself does not so pertain to the main parts of the simple epicyclic gear, therefore applying more such wheels does not have a decisive significance. To increase the number of epicyclic wheels we have two methods at our disposal. When the number of the epicyclic wheels is increased so that each of them remains also separately in direct connection with both central wheels (parallel epicyclic wheels, Fig. 3.), then this increases only the quantity of the transmissible forces, that is, from our point of view has no significance at all. (Otherwise, generally it is customary to apply three epicyclic wheels this having advantages also from viewpoint of fitting.)

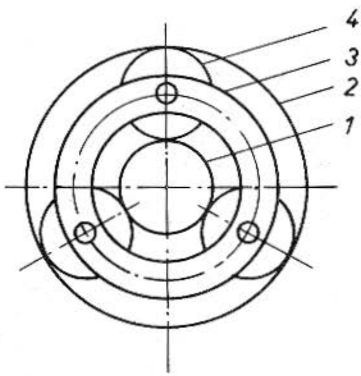


Fig. 3.

If, on the other hand, the number of the epicyclic wheels is increased so that an epicyclic wheel should be in direct connection with one central wheel only, while with the other central wheel comes into contact only by way of a socalled auxiliary wheel, then this will have effect also on the kinematics and dinamics of the simple epicyclic gear (series epicyclic wheels Fig. 4.)

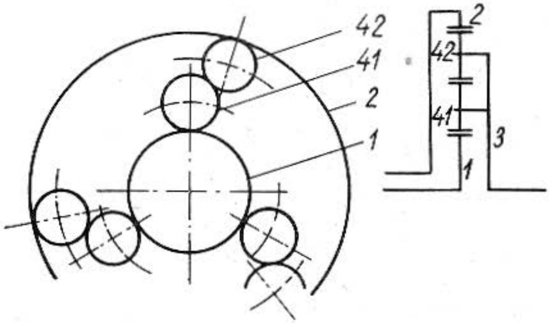


Fig. 4.

Naturally, whatever the number and kind of the epicyclic wheels applied should be, they have to be mounted on the same crank arm.

We can get further variations of the simple epicyclic gear dependently on the circumstance, wether the epicyclic wheels contact the central wheels from the outside or inside (wether in case of applying toothed wheels the gearing is from outside or inside), further, wether both contacts of the epicyclic wheels occur with the same or different diameter.

In Fig. 2. e.g. the epicyclic wheel contacts the central wheel No. 1. with outside connection, the central wheel No. 2. with inside connection. The central wheel is called sun wheel in case of outside connection and annular wheel if the connection is an inside one.

In Fig. 2. the 4 epicyclic wheels join with the same diameter the central wheels No 1 and 2, while in Fig. 5. the diameters are different. The epicyclic gear having two kinds of diameter is the double epicyclic gear.

The simple and double epicyclic wheels, the outside--inside connections can be varied differently, taking also in consideration the possibility of applying an auxiliary epicyclic gear.

All possible variations can be found in Fig. 6. On the left side of the Fig. are the simple epicyclic gear types in which the epicyclic wheels (either simple or double) contact both central wheels. In the epicyclic gears on the right side the epicyclic wheels are to be found by pairs and they contact partly one another, partly one of the central wheels. It is practically superficial to take a further auxiliary epicyclic wheel into consideration.

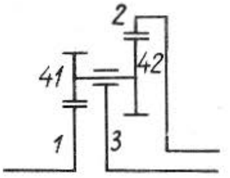


Fig. 5.

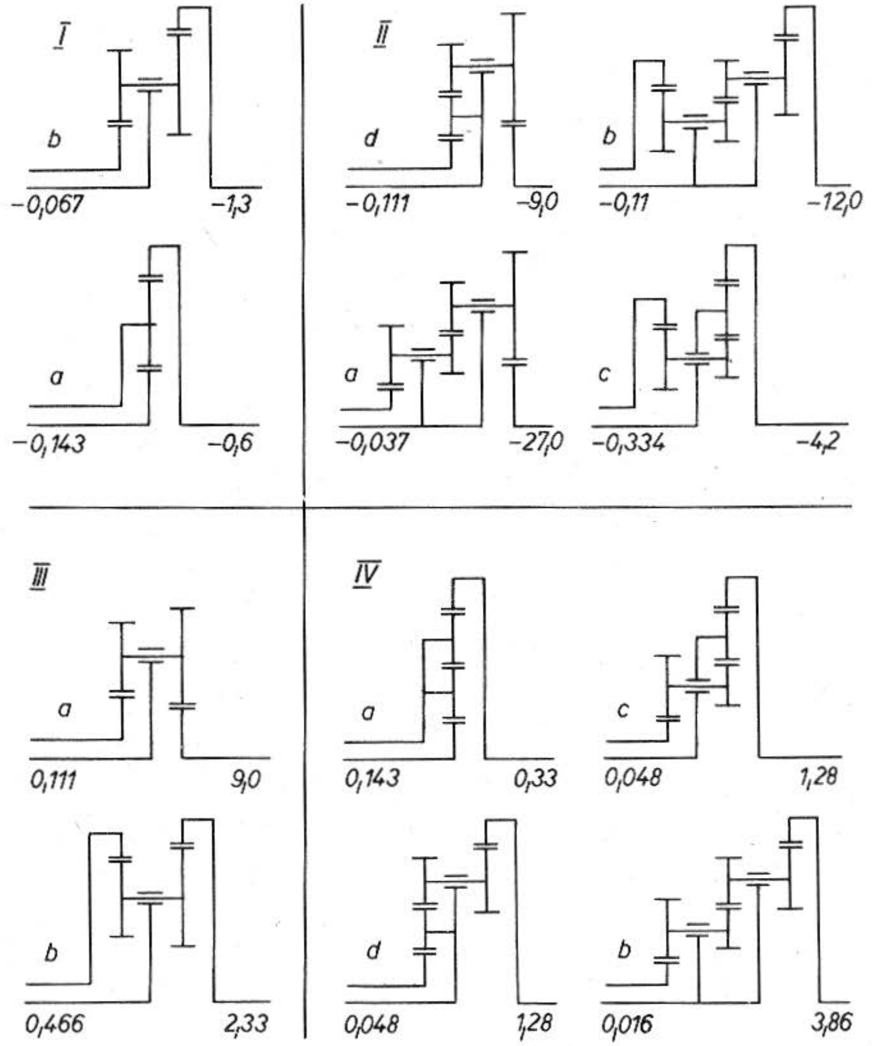


Fig. 6.

It must be noted that bevel epicyclic gears are frequently used, these, however, do not represent a newer type, since every bevel epicyclic gear corresponds to a spur wheel epicyclic gear (Fig. 7.), there is a difference only in the realizable characteristics. So bevel epicyclic gears will not be separately mentioned any more in the following.

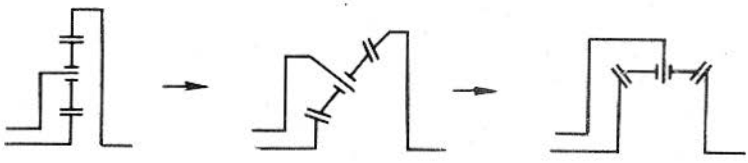


Fig. 7.

The epicyclic gear types shown in Fig. 6 can be summarized in two basic types (Fig. 8 and 9). Any special type can be derived from these two basic types. So e.g. starting from Fig. 8 we get variation I.b. of Fig. 6 taking of the two central wheels the one as sun wheel, and as annular wheel the other. When, moreover, we assume that $D_{41} = D_{42}$ the type I.a. will be arrived at. In the same

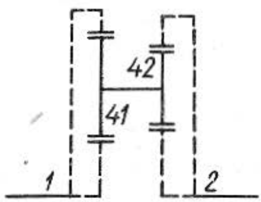


Fig. 8.

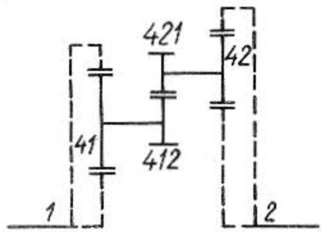


Fig. 9.

way from Fig. 9. can be obtained e.g. the type IV.d. by selecting a sun wheel and an annular wheel if $D_{41} = D_{412}$, resp. the type IV.a. if also $D_{42} = D_{421}$ will be assumed.

22. Kinematical examination fo the simple epicyclic gear

The are known several methods for examining the simple epicyclic gear kinematically, though all of these methods lead to the same result. Of these methods we will select an analytical and a graphical one.

According to the analytical method of Willis (94) the simple epicyclic gear can be characterized by the kinematical basic transmission which is the quotient of the relative angular speeds of the two central wheels referred to the crank arm. Let us choose the symbol b for the kinematical basic transmission, then it can be written that

$$b = \frac{\omega_2'}{\omega_1'} \tag{1}$$

At proceeding to angular speeds, since

$$\omega_1' = \omega_1 - \omega_3 \tag{2}$$

$$\omega_2' = \omega_2 - \omega_3 \tag{3}$$

we get

$$b = \frac{\omega_2 - \omega_3}{\omega_1 - \omega_3} \tag{4}$$

By transposing equation (4) the kinematical basic equation of the simple epicyclic gear will be obtained:

$$b\omega_1 - \omega_2 - (b-1)\,\omega_3 = 0 \tag{5}$$

The basic transmission depends on the type and geometrical dimensions of the epicyclic gear. Its value can be determined in the simplest way in case of the crank arm being at standstill, since then the relative angular speed of the central wheels is identical with the absolute angular speed, i.e. in this case the epicyclic gear may be taken as a simple gear drive the transmission of which can be calculated from the wheeldiameters:

$$b = \frac{\omega_2'}{\omega_1'} = \frac{\omega_2}{\omega_1} = u = \frac{D_1\ D_{42}}{D_2\ D_{41}} \tag{6}$$

or in case of an auxiliary epicyclic wheel:

$$b = -\frac{D_1\ D_{42}\ D_{412}}{D_2\ D_{41}\ D_{421}} \tag{7}$$

The diameters have to be subsituted with a sign, in case of outside connection (gearing) with a positive, by inner connection (gearing) with a negative one. Accordingly, the kinematical basic transmission can also have a positive or negative sign. In Fig. 6 the epicyclic gears were grouped so that in the upper part the types with negative, in the lower part the ones with positive basic transmission can be found.

The magnitude of the basic transmission can not be limitlessly great or small, neither from theoretical, nor from practical point of view. Selection of the basic transmission is limited among others also by that, that exceedingly great difference can not be between the diameters of two wheels engaged to each-other, if only for the danger of undercut. In case of parallel epicyclic gears the vicinity condition must be within view, lest the teeth for two epicyclic wheels should come in connection with each-other, Let us e.g. examine the practically realizable basic transmissions in case of three parallel epicyclic wheels, assuming for the diametrical conditions of the wheels connecting each-other directly or indirectly the moderate values seen in Figs 10 and 11.

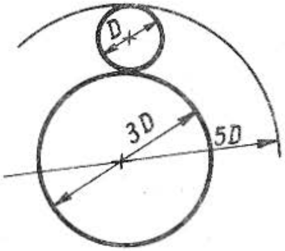


Fig. 10.

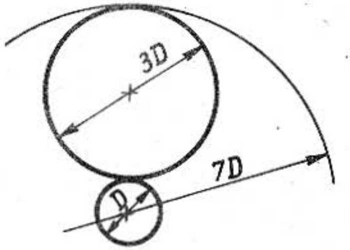


Fig. 11.

From the two Figs can be seen that the diameter-ratio of the wheels with the largest and smallest outside gearing (be it a central or an epicyclic wheel) can be max. 3:1 and the same for the wheels with the smallest ang largest inside gearing min. 1:7.

With this limits the boundaries of all the type-transmissions were calculated and shown in Fig. 6.

The basic transmission-boundaries safely realizable are shown also graphically in Fig. 12 for the sake of clearness. As a matter of course, basic transmissions deviating from these are also realizable, in this case, however, the epicyclic gear has to be controlled with special care from structural viewpoint.

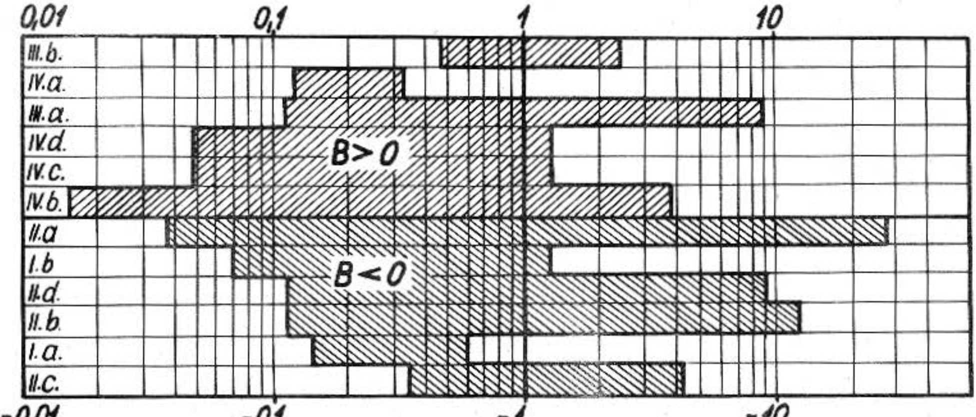


Fig. 12.

The graphical method of Kutzbach (94) is based on the thesis of kinetics, according to which the momentary speed of its any point can be determined if the momentary speed of two points of a rigid body rotating round an unknown central point in the plane, is known. Kutzbach built his method on the epicyclic gear without an auxiliary epicyclic wheel, where the epicyclic wheel may be taken as such a rigid body, since it has a common point with all three members of the epicyclic gear: its central point is fixed to the crank arm, while it contacts the two central wheels slidelessly in a point each on its periphery. The momentary speed of the epicyclic wheel in the three points equals the peripheral speed in that place of just the three members.

Of the three peripheral speeds two pertains to the two degrees of freedom, while the third is given from the former two. Determination is performed graphically (Fig. 13.). Speed is illustrated by vectors. It goes without saying that every peripheral speed vector is perpendicular to the straight line connecting the central points of the epicyclic gear and epicyclic wheel. So the connecting straight line cuts out the unknown third speed vector on the one hand, while on the other, the momentary central point of rotation of the epicyclic wheel.

The angular speed of the epicyclic gear's individual members, resp. the straight pieces proportional to this are cut out by the radii laid across the endpoint of the vectors from the central point of the epicyclic gear on a straight line drawn parallel to the vectors on a place selected deliberately.

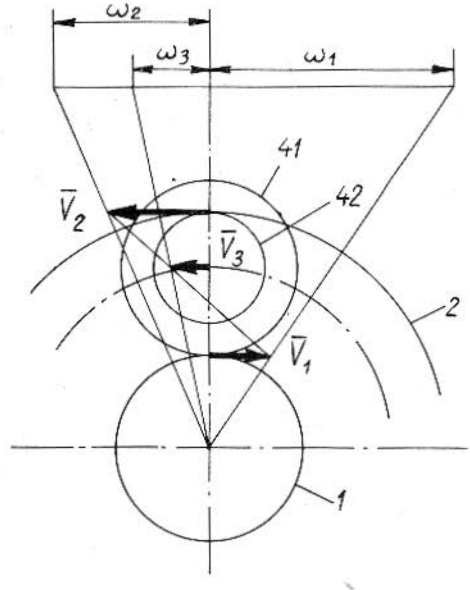


Fig. 13.

As the Kutzbach speedfigure as already mentioned can not be drawn in case of series connected epicyclic wheels, we have developed a method of designing applicable to an epicyclic gear with auxiliary epicyclic wheel.

Let us take as a base for our researches type IV. b (Fig. 14). According to the two degrees of freedom we have taken the vectors V_2 and V_3. Of the two epicyclic wheels let the wheel 42 - 421 be the main epicyclic wheel, so let us refer V_3 to the central point of this. (Wheel 41 - 412 will be, therefore, the auxiliary epicyclic wheel.)

The straight line connecting the endpoints of the two vectors cuts out the momentary point of rotation O_1 of epicyclic wheel 42 - 421, does not present, however, the peripheral speed of the third member - that of the sun wheel in the given case - for it has no common point with it.

Consequently, the endpoint of vector V_1 lies not on the straight line going through the endpoints of vectors V_2 and V_3.

This vector V_1' is naturally also the peripheral speed of wheel 1 but not on the real diameter but on some distance R_1' from the central point.

Corresponding to the linear relation:

$$\frac{\overline{V}_2 - \overline{V}_1'}{R_1' - R_3 - R_{42}} = \frac{\overline{V}_3 - \overline{V}_2}{R_{42}} \tag{8}$$

From this

$$R_1' = R_3 + R_{42} \, \frac{\overline{V}_3 - \overline{V}_1'}{\overline{V}_3 - \overline{V}_2} \tag{9}$$

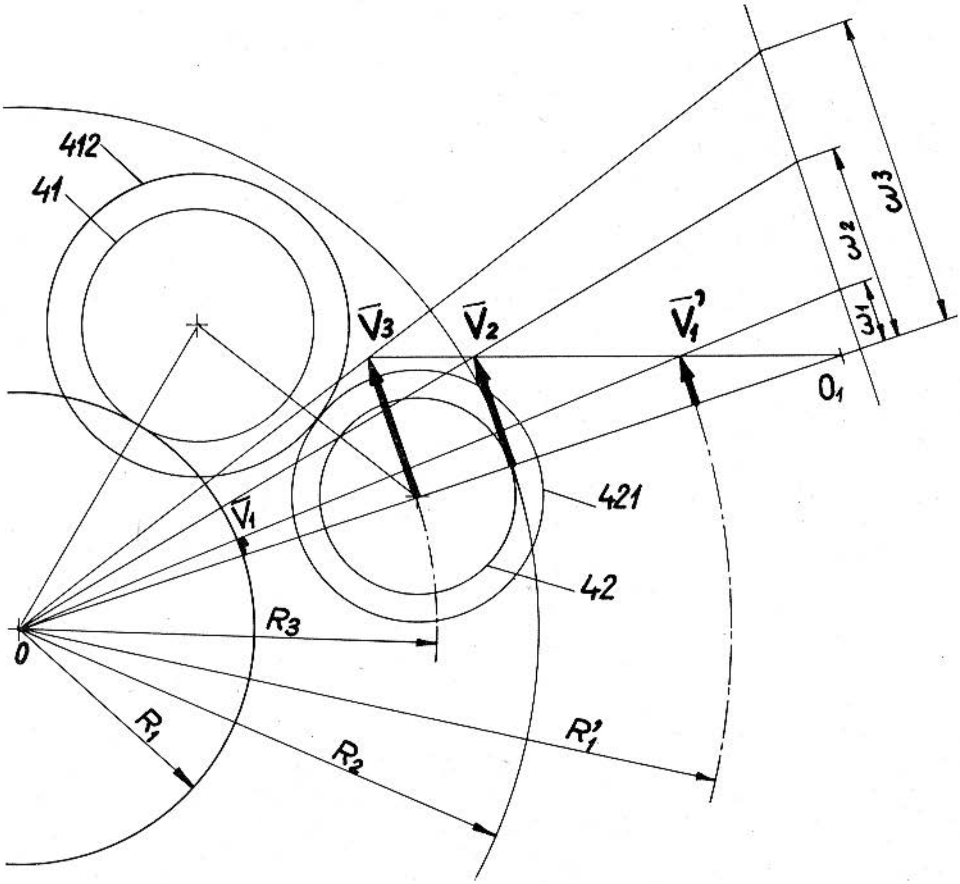


Fig. 14.

Peripheral speeds expressed by angular speeds:

$$\bar{V}_1' = R_1' \omega_1 \tag{10}$$

$$\bar{V}_2 = (R_3 + R_{42}) \omega_2 \tag{11}$$

$$\bar{V}_3 = R_3 \omega_3 \tag{12}$$

These substituted, after reducing the equation we obtain

$$R_1' = R_3 \frac{(R_3 + R_{42}) \quad (\omega_3 - \omega_2)}{R_3 \omega_3 - (R_3 + R_{42}) \omega_2 + R_{42} \omega_1} \tag{13}$$

Corresponding to the two degrees of freedom let us assume e.g.:

$$\omega_3 = 0$$

$$\omega_1 = 1$$

We obtain that

$$\omega_2 = \frac{R_1}{R_{41}} \frac{R_{412}}{R_{421}} \frac{R_{42}}{R_3 + R_{42}} \tag{14}$$

Substituting the angular speeds the result will be:

$$R_1' = \frac{R_3}{1 - \frac{R_{41}}{R_1} \frac{R_{421}}{R_{412}}} \tag{15}$$

This result, however, does not satisfy us since we are in quest of a pure graphical method.

In Fig. 15 the central points and contacting points of the wheels are shown again. Let us draw a straight line through the contacting points of auxiliary epicyclic wheel 41 - 412. This straight line intersects in point Q the straight line $\overline{OO_2}$. We should now draw a straight line parallel to straight line $\overline{OO_2}$ through the contacting point of wheel 1. The straight line sections obtained in this way will be marked a, b and c.

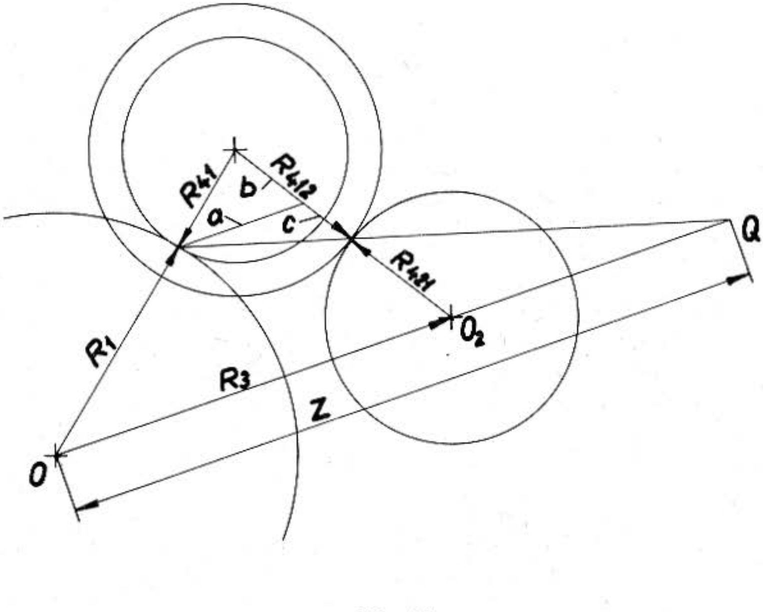


Fig. 15.

On base of the similarity of triangles can be writen:

$$\frac{a}{Z-R_3} = \frac{R_{412}-b}{R_{421}} \tag{16}$$

Expressing Z of this:

$$Z = R_3 + \frac{a\,R_{421}}{R_{412}-b} \tag{17}$$

In the expression (17) only a and b are unknown. It may be written also on base of the similarity of triangles:

$$\frac{a}{R_3} = \frac{R_{41}}{R_1+R_{41}} \tag{18}$$

resp.

$$R_{41}\,\frac{R_3}{R_1+R_{41}} \tag{19}$$

and

$$\frac{b}{R_{412}+R_{421}} = \frac{R_{41}}{R_1+R_{41}} \tag{20}$$

resp.

$$b = R_{41}\,\frac{R_{412}+R_{421}}{R_1+R_{41}} \tag{21}$$

Substituting the expressions got for a and b into formula Z, we obtain after reducing:

$$Z = \frac{R_3}{1-\frac{R_{41}\,R_{421}}{R_1\,R_{412}}} \tag{22}$$

Comparing formulae (15) and (22) it is evident that $Z = R_1'$.

I.e. radius R' can be determined also graphically so that with the straight line drawn through the contacting points of the auxiliary epicyclic gear we cut it out of the straight line going through the central points of the main epicyclic wheel and central wheel.

After these, the process of the graphical method will be the following:

Of the two epicyclic wheel in contact with each other one will be selected as main epicyclic wheel and design work will be performed for this according to Kutzbach, though with the modification that the diameter of the central wheel not contacting the main epicyclic wheel will be taken as 2R' (independently of the real diameter) the place of which is cut out by the straight line going through the contacting points of the auxiliary epicyclic wheel from the straight line laid through the main epicyclic wheel's central point.

The simplicity of the process will be shown on an example.

Let us take an epicyclic gear of the type IV.d. at which $R_{41} = R_{412}$, i.e. one of the epicyclic wheels is not double. As a base for our design work, at first this epicyclic wheel will be selected and so wheel 42 - 421 will be the auxiliary epicyclic wheel (Fig. 16).

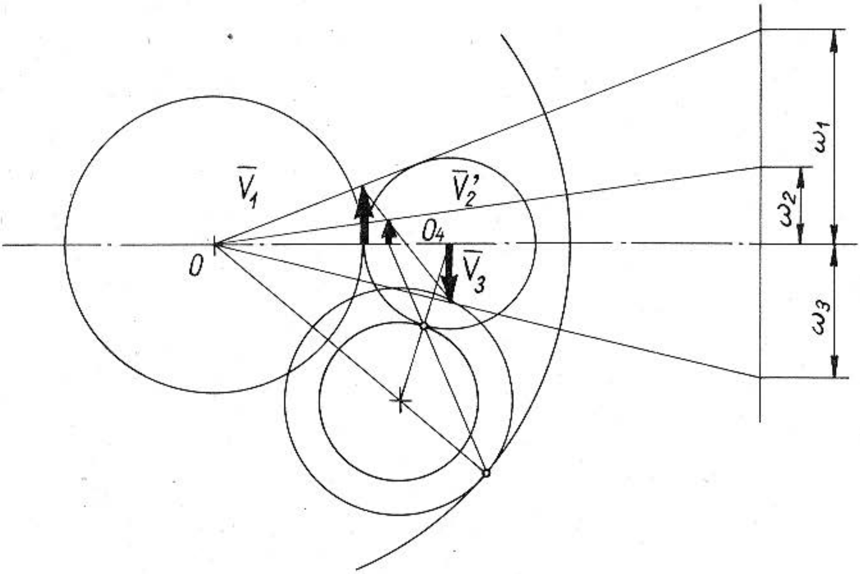


Fig. 16.

Central wheel 2 does not contact directly the epicyclic wheel under examination. The straight line drawn through the contacting points of the auxiliary epicyclic wheel cuts out radius R_2' of central wheel 2.

Let us take e.g. angular speeds ω_1 and ω_2. The peripheral speed vectors V_1 and V_2 will be drawn. (Vector V_2 can also be drawn on the real periphery of wheel 2, this, however, has no specific significance). The straight line connecting the end-points of vectors V_1 and V_2 cuts out the end-point of vector V_3. Projecting this onto the straight line of the angular speeds we obtain the length of straight line proportional to w_3.

The same result must be arrived at when performing the designing at this same epicyclic gear for epicyclic wheel 42 - 421 (Fig. 17).

At the second case may be observed that the diameter of the auxiliary epicyclic wheel - not being a double wheel - has no influence whatever on the epicyclic gear's kinematics.

This can be read off also from formula R. The formula was written for the case when epicyclic wheel 41 - 412 has been considered as an auxiliary epicyclic wheel, so if

$$R_{41} = R_{412} \quad \text{then} \tag{23}$$

the formula is:

$$R' = \frac{R_3}{1 - \frac{R_{421}}{R_1}} \tag{24}$$

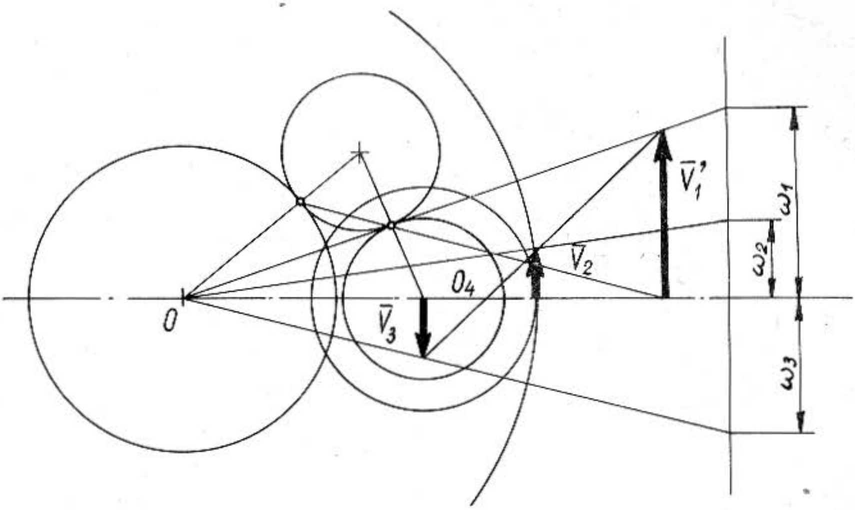


Fig. 17.

i.e. R' does not depend on the dimensions of the auxiliary epicyclic wheel. But if this dependence does not exist, then it can be also indefinitely great. This circumstance renders the designing still more simple, since plotting of the simple (not double!) auxiliary epicyclic wheel will be superfluous. Radius R' can also be obtained by the tangent drawn to the contacting central wheel and to the main epicyclic wheel. This tangent, namely, corresponds to the interwedged indefinite radius-epicyclic whell' periphery (Fig. 18).

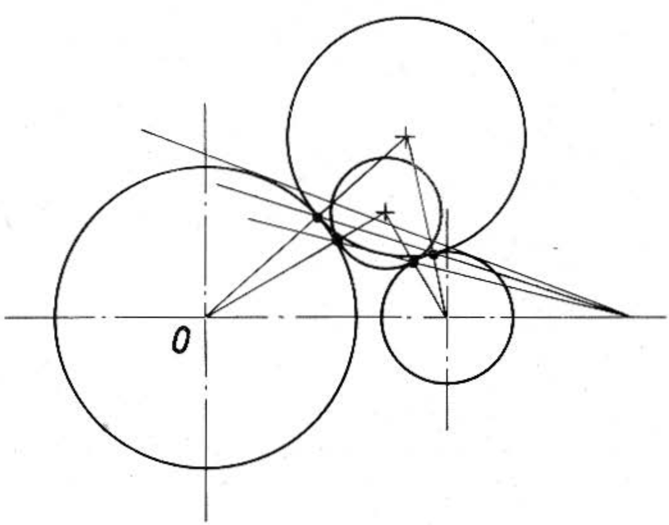


Fig. 18.

Since, however, the simple epicyclic wheel can be taken in consideration either as main or auxiliary epicyclic wheel deliberately, it is obvious, that the above said can be generalized: in the case when in an epicyclic gear of the epicyclic wheels contacting each other one is not a double wheel then its dimensions have no influance on the kinematics of the epicyclic gear. If none of the two are double wheels, then the diameters of both epicyclic wheels - from viewpoint of the epicyclic gear's outside kinematics - can be arbitrary. As the result of this, only the dimensions of the two central wheels and of the double epicyclic wheel have significance. The same may be read off from formulae (6) and (7).

23. Dynamical examination of the simple epicyclic gear

Since the simple epicyclic gear has three such members which can be led out, the three torques having effect on it - uniform rotation supposed - are always in equilibrium:

$$M_1 + M_2 + M_3 = 0 \tag{25}$$

Since, further, in case of $w_3 = 0$ the epicyclic gear becomes transformed to a simple gearing mechanism, for this case can be written:

$$\frac{M_2}{M_1} = -\frac{\omega_1}{\omega_2} = -\frac{1}{b} \tag{26}$$

wherefrom

$$M_2 = -\frac{1}{b} M_1 \tag{27}$$

From equation (25) follows that

$$M_3 = -M_1 - M_2 = \left(\frac{1}{b} - 1\right) M_1 \tag{28}$$

The geometrical dimensions being constant, it follows that the relations (27) and (28) are valid for all w_3 values. It is advised to write these two relations in proportions:

$$M_1 : M_2 : M_3 = 1 : -\frac{1}{b} : \left(\frac{1}{b} - 1\right) \tag{29}$$

It can be seen, that contrarily to the kinematical degree of freedom, assumption of a single torque already makes determined the other torques too.

From the expression (29) - after multiplication with the adequate angular speeds - we obtain the proportions characteristic for the performance conditions:

$$P_1 : P_2 : P_3 = \omega_1 : \left(-\frac{1}{b}\right) \omega_2 : \left(\frac{1}{b} - 1\right) \omega_3 \tag{30}$$

3. THE COMPLEX AND THE CONJUNCT EPICYCLIC GEARS

It occurs in practice that an epicyclic gear has three members (led out shafts), these, however, do not conform to the conditions written on page 3 according to which the three members of the simple epicyclic gear are: two central wheels and a crank-arm. In Fig.19 e.g. such an epicyclic gear can be seen all the three led out shafts of which pertain to central wheels. These are the complex epicyclic gears since they actually consist of two or more simple epicyclic gears. The type in Fig.19 e.g. consists of a simple epicyclic gear type III a and of another of type I a. Frequently occurs that in both simple epicyclic gears can be found members of the same dimensions and function and just these will be connected. In such case these members can be united which leads to the simplifications of the structure. In Fig.20 e.g. such a so-called conjuct epicyclic gear is shown which as a matter of fact corresponds to the complex epicyclic gear shown in Fig.19, in the specific case if $D_2^M = D_1^N$ and $D_{42}^M = D_4^N$.

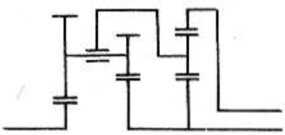


Fig.19

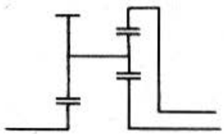


Fig.20

It is obvious that the complex and conjunct epicyclic gears can have innumerable variations, this following from the fact that the simple epicyclic gears may be combined in a great many manners.

Here only some examples of the conjunct epicyclic gears will be presented to which, later on, references will be made.

The conjunct epicyclic gear shown in Fig.21 consist of two simple epicyclic gears of the type IIIa.

The variation presented in Fig.22 is an example of the level-wheel epicyclic gear. Strictly speaking, here Figs 19 and 20 repeat themselves.

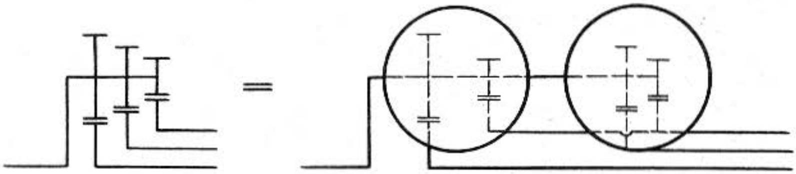


Fig.21

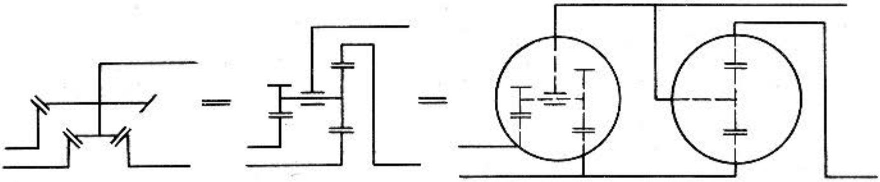


Fig. 22.

The complex epicyclic gear shown in Fig. 23 may be separated also to types I a and IV a in a way similar to Fig. 22, however, for the sake of presenting the variability, here the simple epicyclic gears I b and IV a were taken as bases.

Fig. 24 is developed from Fig. 21 by using a further simple epicyclic gear. That is, here outside of the two III a types a type I a also was united.

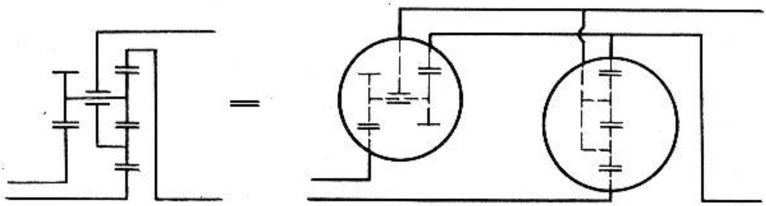


Fig. 23.

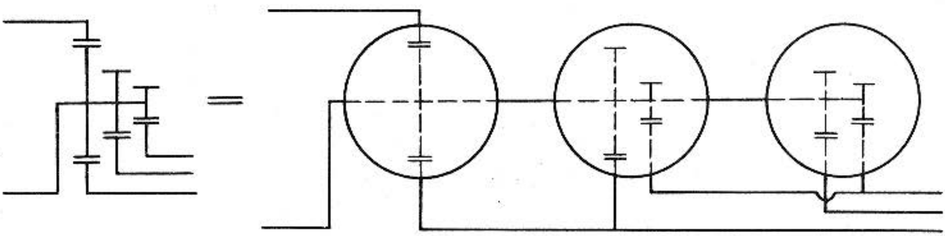


Fig. 24.

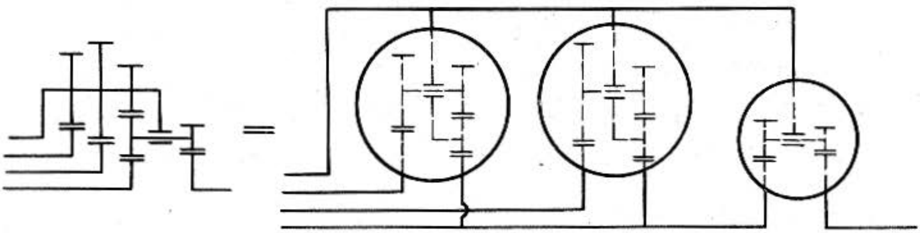


Fig. 25.

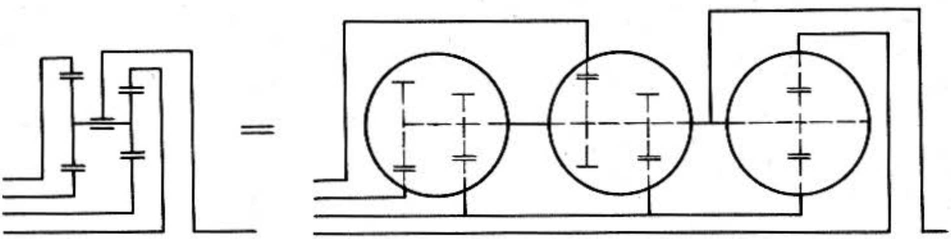


Fig. 26.

II. THE COMPLEX EPICYCLIC GEAR

1. GENERALIZATION OF THE SIMPLE AND COMPLEX EPICYCLIC GEARS

11. Purpose of the generalization

As could be seen in the foregoing chapter, the kinematical basis transmission b determines seemingly unambigously not only the kinematical (5) but also the dynamical (29), (30) conditions of an epicyclic gear. On taking in use a certain epicyclic gear this is really so since then the function of the epicyclic gear - members is already known. The epicyclic gear-members can in general perform different tasks resp. fulfill different roles in the power- or motion transmission. With other words the distribution of the individual functions between the three members can be optional. The role of an epicyclic gear-member depends on the manner of its binding in.

It would considerably simplify the analytical examinations if we could leave out of attention the concrete naming of the epicyclic gear-members and, regulating the manner of binding in, it could be based only on the function of the epicyclic gear-members.

12. Generalization of the simple epicyclic gear

Consequently to the above we propose the generalization of the Willis kinematical basic transmission, the introducing of the general kinematic basic transmission, with the simultaneous determining of the connection between the two basic transmissions.

For this purpose we take for base instead of the three members of the epicyclic gear the three led out shafts with the understanding that the three members and the three shafts can be connected in any variation selected optionally. In fact, 6 variations are feasible to bind in the epicyclic gear-members.

For the epicyclic gear shafts the Willis formula can also be written:

$$B = \frac{\omega'_{II}}{\omega'_{I}} = \frac{\omega_{II} - \omega_{III}}{\omega_{I} - \omega_{III}} \tag{31}$$

where

B — is the general kinematical basic transmission

$\omega_I, \omega_{II}, \omega_{III}$ — are the angular speeds of the shafts marked with indices I, II and III.

Table 1 contains the connection between B and b according to the six variations of binding in. Indices 1 and 2 pertain also here, as ever, to the central wheel, while index 3 to the crank arm.

As a result of the generalization we obtained such a characteristic number, which characterized the epicyclic gear unambigously, independently of its type, geometrical dimensions and the manner of its binding in. It is obvious that in case of the values B being identical the characteristics of the different epicyclic gears are also identical. It may occur therefore that two epicyclic gears with two different b basic transmissions - in case of different binding in - will behave uniformly or two identical epicyclic gears will show quite different character, in consequence of their different binding in.

Thus in the following the general kinematical basic transmission will be used, and accordingly, the epicyclic gears will be illustrated with a mere simple circle.

Similary to formula (5) the general kinematical basic equation can also be written:

$$B\omega_{I} - \omega_{II} - (B-1)\omega_{III} = 0 \tag{32}$$

or, dispensing with the deduction, the torque proportions:

$$M_{I} : M_{II} : M_{III} = 1 : -\frac{1}{B} : \left(\frac{1}{B} - 1\right) \tag{33}$$

resp. the performance proportions

$$P_{I} : P_{II} : P_{III} = \omega_{I} : \left(-\frac{1}{B}\right)\omega_{II} : \left(\frac{1}{B} - 1\right)\omega_{III} \tag{34}$$

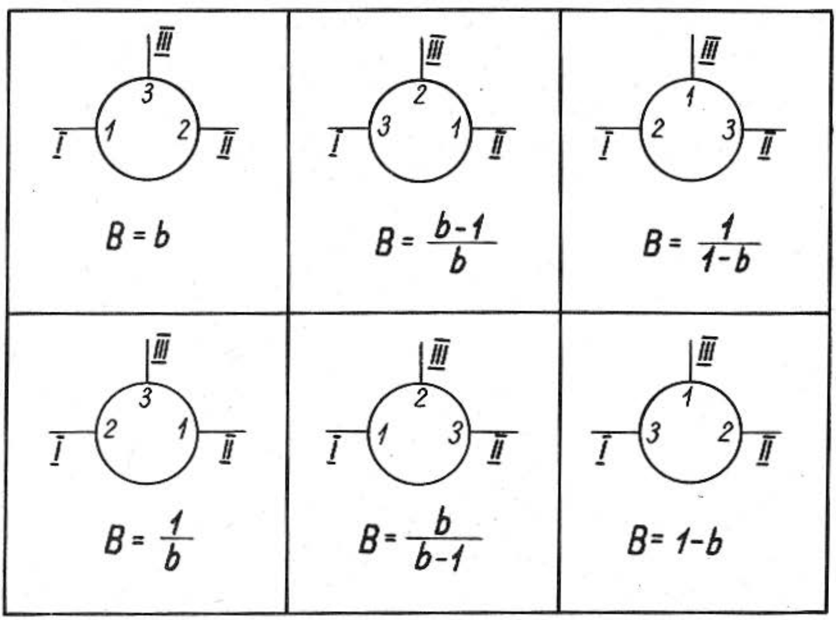


Table 1.

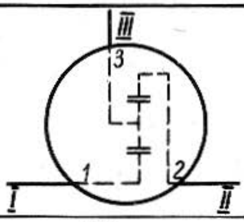
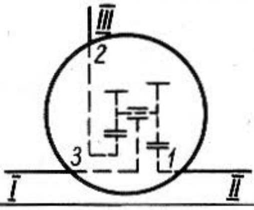
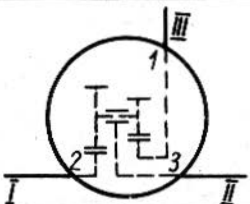
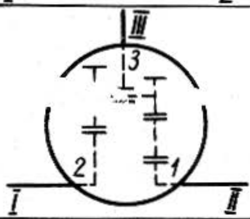
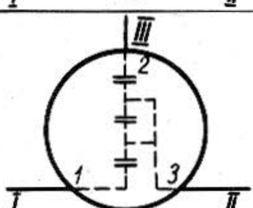
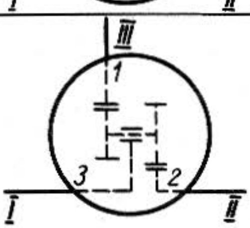
	$r_1 = 4$ $r_2 = -16$	$b = \frac{r_1}{r_2} = -025$ $B = b = -0,25$
	$r_1 = 6$ $r_2 = 6,57$ $r_{41} = 3$ $r_{42} = 2,43$	$b = \frac{r_1}{r_{41}} \frac{r_{42}}{r_2} = 0,8$ $B = \frac{b-1}{b} = -0,25$
	$r_1 = 6$ $r_2 = 2,58$ $r_{41} = 3$ $r_{42} = 6,42$	$b = \frac{r_1}{r_{41}} \frac{r_{42}}{r_2} = 5$ $B = \frac{1}{1-b} = -0,25$
	$r_1 = 6$ $r_2 = 2$ $r_{41} = 3$ $r_{42} = 4$	$b = \frac{r_1}{r_{41}} \quad \frac{r_{42}}{r_2} = -4$ $B = \frac{1}{b} = -0,25$
	$r_1 = 4$ $r_2 = 20$	$b = - \frac{r_1}{r_2} = 0,2$ $B = \frac{b}{b-1} = -0,25$
	$r_1 = -18$ $r_2 = -20,6$ $r_{41} = 6$ $r_{42} = 8,6$	$b = \frac{r_1}{r_{41}} \quad \frac{r_{42}}{r_2} = 1,25$ $B = 1-b = -0,25$

Table 2.

13. Generalization of the complex epicyclic gear

After the generalization of the simple epicyclic gear the same for the complex one seems to be a task easy enough. Before proceeding, however, we must come to an understanding regarding the general type of the complex epicyclic gear. We propose to accept as this general type the connection shown in Fig. 27, where the two II shafts resp. two III shafts of M and N simple epicyclic gears are connected. Thus the basic type of the complex epicyclic gear has four shafts in all. Shafts I and II are connected always only with a simple epicyclic gear-member, the two III, i.e. III_1 and III_2 shafts always with two epicyclic gear-members and namely with one member of the two simple epicyclic gears each.

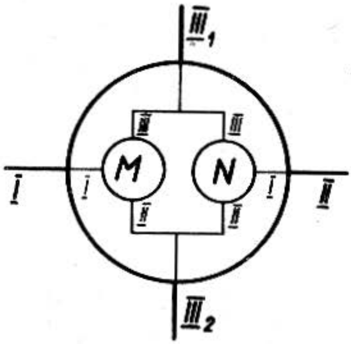


Fig. 27.

Considering that at the two simple epicyclic gears with two degrees of freedom two shafts are connected, the complex epicyclic gear also remains a two degree of freedom mechanism in spite of its four shafts. Of this follows that for the kinematical basic transmission we obtain here two values, corresponding to the fact that the general kinematical basic equation can be written for the I-II-III_1 and I-II-III_2 shafts.

In order to determine this let us write the general kinematical basic equation of the two simple epicyclic gear:

$$\left.\begin{aligned} B^M \omega_{\mathrm{I}}^M - \omega_{\mathrm{II}}^M - (B^M-1)\,\omega_{\mathrm{III}}^M &= 0 \\ B^N \omega_{\mathrm{I}}^N - \omega_{\mathrm{II}}^N - (B^N-1)\,\omega_{\mathrm{III}}^N &= 0 \end{aligned}\right\} \tag{35}$$

Since

$$\omega_{\mathrm{II}}^N = \omega_{\mathrm{II}}^M$$

$$\omega_{\mathrm{I}}^M = \omega_{\mathrm{I}}$$

$$\omega_{\mathrm{I}}^N = \omega_{\mathrm{II}}$$

$$\omega_{\mathrm{III}}^M = \omega_{\mathrm{III}}^N = \omega_{\mathrm{III}_1}$$

therefore

$$\left.\begin{aligned} B^M \omega_{\mathrm{I}} - \omega_{\mathrm{II}}^M - (B^M-1)\,\omega_{\mathrm{III}_1} &= 0 \\ B^N \omega_{\mathrm{II}} - \omega_{\mathrm{II}}^M - (B^N-1)\,\omega_{\mathrm{III}_1} &= 0 \end{aligned}\right\} \tag{36}$$

After eliminating ω_{II}^{M} the general kinematical basic equation of the complex epicyclic gear will be obtained for shafts I II III_1

$$\frac{B^M}{B^N}\omega_I - \omega_{II} - \left(\frac{B^M}{B^N} - 1\right)\omega_{III_1} = 0 \tag{37}$$

resp. comparing the general kinematical basic transmission to equation (32):

$$B_1 = \frac{B^M}{B^N} \tag{38}$$

When taken in consideration that

$$\omega_{II}^{M} = \omega_{II}^{N} = \omega_{III_2} \quad \text{and}$$

$$\omega_{III}^{M} = \omega_{III}^{N}$$

then according to the former deduction for shafts $I-II-III_2$ will be obtained the general kinematical basic equation:

$$\frac{B^M}{B^N}\frac{(B^N-1)}{(B^M-1)}\omega_I - \omega_{II} - \frac{B^N - B^M}{B^N\,(B^M-1)}\omega_{III_2} = 0 \tag{39}$$

resp. the basic transmission:

$$B_2 = \frac{B^M(B^N-1)}{B^N(B^M-1)} \tag{40}$$

The dynamical conditions of the complex epicyclic gear can also be examined by the (33) and (34) formulae, taken in consideration, as a matter of course, formulae (38) and (40) too. Thus it can be written:

$$M_I : M_{II} : M_{III_1} = 1 : -\frac{1}{B_1} : \left(\frac{1}{B_1} - 1\right) \tag{41}$$

resp.

$$M_I : M_{II} : M_{III_2} = 1 : -\frac{1}{B_2} : \left(\frac{1}{B_2} - 1\right) \tag{42}$$

and

$$P_I : P_{II} : P_{III_1} = \omega_I : \left(-\frac{1}{B_1}\right)\omega_{II} : \left(\frac{1}{B_1} - 1\right)\omega_{III_1} \tag{43}$$

resp.

$$P_I : P_{II} : P_{III_2} = \omega_I : \left(-\frac{1}{B_2}\right)\omega_{II} : \left(\frac{1}{B_2} - 1\right)\omega_{III_2} \tag{44}$$

2. EPICYCLIC GEARS WITH TWO DEGREES OF FREEDOM

The simple or the complex epicyclic gear (epicyclic drive gear) is originally always a mechanism with two degrees of freedom.

When using it in practice, sometimes just this is utilized e.g. for summarizing, separating, equalizing resp. distributing two rotational motions resp. performances more or less independent of each other.

In the first case we are dealing with double drive-in, in the second with double drive-out epicyclic gears.

There is double drive-in e.g. in taxameters where the epicyclic gear has to summarize the motions of the cardan shaft and that of the shaft of the time-metering watch. The most characteristic example for the double drive-out is the differential gear of motor vehicles.

For the kinematical resp. dynamical examination of double drive epicyclic gears the kinematical basic equation resp. the torque and performance proportions can be used directly. For just this reason and for their relatively narrow utilization-fields we dispense with dealing with them any more.

3. EPICYCLIC GEARS WITH ONE DEGREE OF FREEDOM

31. Simple epicyclic gear with one degree of freedom

In the practice one degree of freedom structures are needed in most cases and this is so at the change-speed gears of motor vehicles too, where unambigous kinematical and dynamical conditions have to be predominant between the in - and outgoing shafts. Therefore, the epicyclic gear with two degrees of freedom must be transformed to a unit of one degree of freedom. There are two ways for this. The one way is the fastening of one shaft of the epicyclic gear - and for the sake of unambigousity always of the III shaft, i.e. rendering it connected to the base (Fig. 28). The other way is to fasten this same shaft to another shaft - in the following uniformly to the II shaft (Fig. 29). We propose to name the epicyclic gear simply fastened epicyclic gear in the first case, and transfastened epicyclic gear in the other. For fastening resp. transfastening the shafts as connecting elements any mechanical, hydraulic, electric, pneumatic, etc. power transmission structures (e.g. clutches, torque converters, etc.) can be used. If the simple epicyclic gear is completed with some kind of connecting element, the assembly is called complex epicyclic gear.

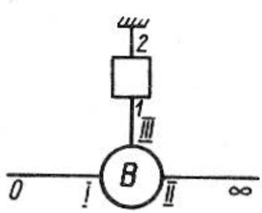


Fig. 28.

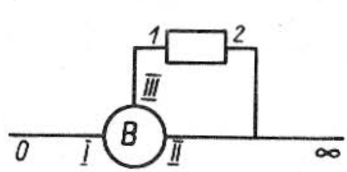


Fig. 29.

In Fig. 28 and 29 the connecting elements are marked by a square. The shaft of the connecting element located towards the O shaft is marked with a 1 index, while its other shaft with a 2 index.

Both at the simply fastened and the transfastened epicyclic gears after all two shafts remained. Let us mark the shafts of the complex epicyclic gear with O and ∞ indices, we use the O index uniformly fromly from shaft I, the ∞ index from shaft II. In the one case we are dealing with straight drive and the transfastened epicyclic gear is called then prefastened epicyclic gear, in the other case we can speak of reversed drive and back-fastened epicyclic gear. For marking the incoming shaft we use the letter x and for the out--going shaft the latter y, whenever such distinction is needed (Fig. 30),

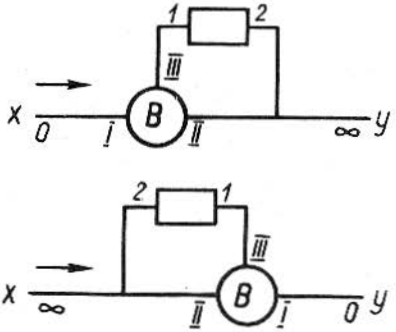


Fig. 30.

Of course, not only one connecting element can be used. Theoretically we may insert a connecting element in all shafts. In Fig. 31 the simply fastened, in Fig. 32 the transfastened epicyclic gear is shown with all the imaginable locations of the connecting elements marked. The possibility of inserting in a single location also two connecting elements - e.g. a mechanical and a hydraulic one - in series connection with each-other, is not, however, indicated here. Otherwise, for marking the connecting element we use the index of that shaft into which it is inserted.

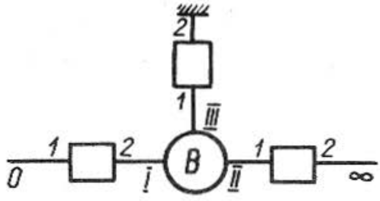


Fig. 31.

Always that shaft of the connecting element is marked with index 1 which is nearer to the shaft O.

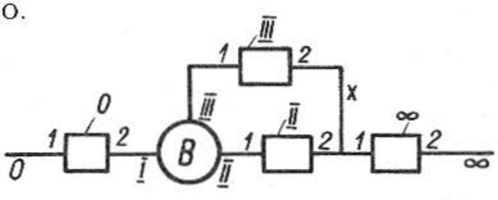


Fig. 32.

32. Complex epicyclic gear with one degree of freedom

To fasten one degree of freedom of the complex epicyclic gear three ways are available. The first two ways are identical to the transformation of the simple epicyclic gear, i.e. one of the two shafts III - further uniformly shaft $\overline{III}_2$ - has to be simply fastened or transfastened. In such case shaft $\overline{III}_1$ runs freely (Fig. 33 and 34.).

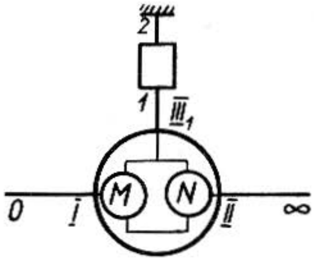


Fig. 33.

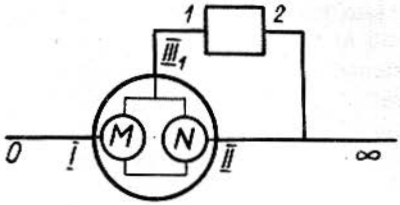


Fig. 34.

The third way is the connection of two of the four shafts and of the remaining two one will be the in-coming and one the out-going shaft. We propose to name such an epicyclic gear crossfastened epicyclic gear. The crossfastened epicyclic gear has two varieties. In Fig. 35 the straight crossfastened epicyclic gear can be seen, where two III shafts are crossconnected, in Fig. 36 the inverted crossconnected epicyclic gear, where shafts I and II are crossconnected.

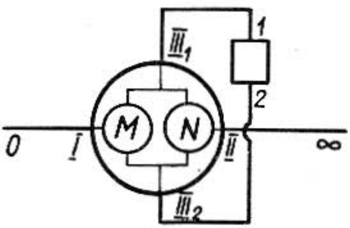


Fig. 35.

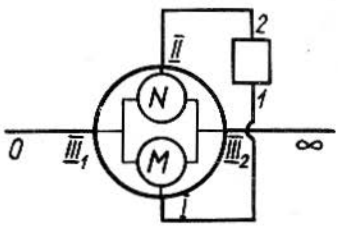


Fig. 36.

Naturally, here also more connecting elements can be inserted, moreover, within the complex epicyclic gear, at the connection of the two simple epicyclic gears, also can a connecting element be used.

In case of using a connecting element within the complex epicyclic gear, as a matter of course, the complex epicyclic gear's kinematical basic equation also will be changed. Dispensing with the deduction we present here the kinematical basic equations valid in this case:

$$B^M i^N_{\mathrm{II}}\; i^M_{\mathrm{II}}\, \omega_{\mathrm{I}} - B^N \omega_{\mathrm{II}} - \left[(B^M-1)\,\frac{i^M_{\mathrm{II}}\; i^N_{\mathrm{II}}}{i^M_{\mathrm{III}}} - (B^N-1)\, i^N_{\mathrm{III}}\right]\omega_{\mathrm{III}_1} = 0 \qquad (45)$$

respectively

$$B^M(B^N-1)\, i^M_{\mathrm{III}}\; i^N_{\mathrm{III}}\, \omega_{\mathrm{I}} - B^N(B^M-1)\,\omega_{\mathrm{II}} - \left[(B^N-1)\,\frac{i^M_{\mathrm{III}}\; i^N_{\mathrm{III}}}{i^M_{\mathrm{II}}} - (B^M-1)\, i^N_{\mathrm{II}}\right]\omega_{\mathrm{III}_2} = 0 \qquad (46)$$

and the torque proportions as well:

$$M_{\bar{I}} : M_{\bar{II}} : M_{\bar{III}_1} = 1 : \left[k^M_{\bar{III}}\left(\frac{1}{B^M} - 1\right) - \frac{k^M_{\bar{II}}\, k^N_{\bar{II}}}{k^N_{\bar{III}}}\ \frac{1 - B^N}{B^M} - 1 \right] : \left[k^M_{\bar{II}}\left(1 - \frac{1}{B^M}\right) - \frac{k^M_{\bar{II}}\, k^N_{\bar{I}}}{k^N_{\bar{III}}}\ \frac{B^N - 1}{B^M} \right] \tag{47}$$

respectively

$$M_{\bar{I}} : M_{\bar{II}} : M_{\bar{III}_2} = 1 : \left[\frac{k^M_{\bar{III}}\, k^N_{\bar{III}}}{k^N_{\bar{I}}}\ \frac{B^M - 1}{B^M(B^N - 1)} - \frac{k^M_{\bar{I}}}{B^M} - 1 \right] : \left[\frac{k^M_{\bar{I}}}{B^M} - \frac{k^M_{\bar{III}}\, k^N_{\bar{III}}}{k^N_{\bar{I}}}\ \frac{B^M - 1}{B^M(B^N - 1)} \right] \tag{48}$$

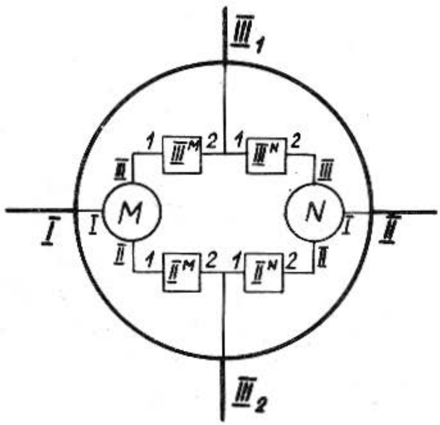


Fig. 37.

In Figs. 38–41 we present the simply fastened, the transfastened and the two kinds of crossfastened epicyclic gears, showing all the possible connecting elemnts.

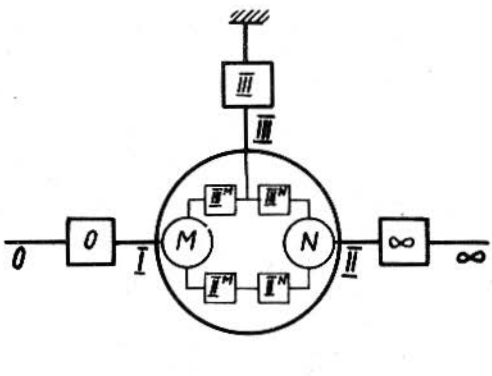


Fig. 38.

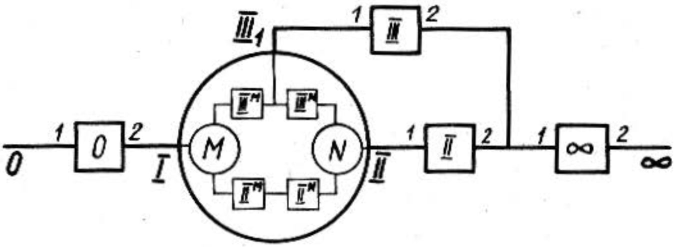


Fig. 39.

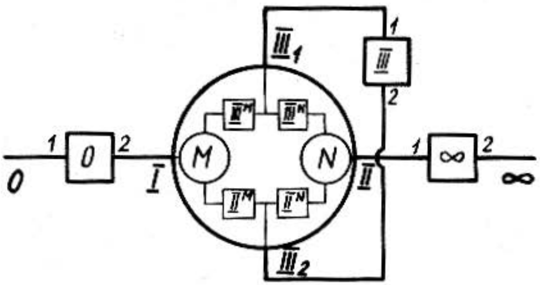


Fig. 40.

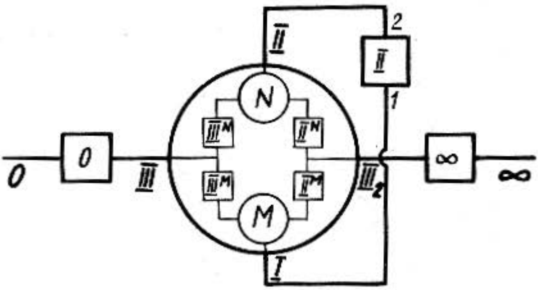


Fig. 41.

4. KINEMATICAL CONDITIONS OF COMPLEX EPICYCLIC GEARS

41. Kinematical conditions of the simply fastened complex epicyclic gear

411. Kinematical gear transmission of the simply fastened simple epicyclic gear.

From viewpoint of the change-speed gear the kinematical conditions of complex epicyclic gears are completely well characterized by the kinematical gear transmission. Therefore in the following only this will be determined.

The number of the possible connecting elements at the simply fastened simple epicyclic gear is three (Fig. 31).

The connecting elements are kinematically characterized by their transmissions:

$$i_0 = \frac{\omega_I}{\omega_0} \tag{49}$$

$$i_\infty = \frac{\omega_\infty}{\omega_{II}} \tag{50}$$

$$i_{III} = \frac{\omega_X}{\omega_{III}} \tag{51}$$

Since the base always stands, therefore $i_{III} = 0$

Instead of this, for the characterizing of the third connecting element we compare the angular speeds of the epicyclic gear's III and I shaft:

$$i_{III\,I} = \frac{\omega_{III}}{\omega_I} \tag{52}$$

Expressing from (49), (50) and (52) the angular speeds of the simple epicyclic gear's shafts and substituting them in the general kinematical basic equation of the simple epicyclic gear, we obtain:

$$B\, i_0 \omega_0 - \frac{\omega_\infty}{i_\infty} - (B-1)\, i_{III\,I}\, i_0\, \omega_0 = 0 \tag{53}$$

From this the transmission of the simply fastened complex epicyclic gear can be expressed:

$$\frac{\omega_\infty}{\omega_0} = i_0\, i_\infty \left[B - (B-1)\, i_{III\,I} \right] \tag{54}$$

Connecting elements i_0 and i_∞ are series connected to the epicyclic gear. The simply fastened complex epicyclic gear's simple case will be obtained when there are no series connected connecting elements (Fig. 28) i.e.

$$i_0 = i_\infty = 1$$

Then

$$\frac{\omega_\infty}{\omega_0} = B - (B-1)\, i_{III\,I}$$

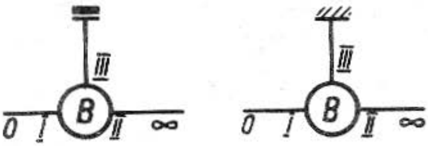


Fig. 42.

A widely used variety of this type is the epicyclic gear simply fastened by clutch (brake). See Fig. 42. In case of closed brake (see right side of Fig.) the transmission

$$\frac{\omega_\infty}{\omega_0} = B \tag{55}$$

since

$$\omega_{III\,I} = 0$$

412. <u>Kinematical gear transmission of the simply fastened complex epicyclic gear</u>. In case of simply fastening shaft $\underline{III}_1$ of the complex epicyclic gear (Fig. 38), after substitutions, the basic equation is:

$$B^M i_{II}^M i_{II}^N i_0 \omega_0 - B^N \frac{\omega_\infty}{i_\infty} - \left[(B^M - 1) \frac{i_{II}^M \, i_{II}^N}{i_{III}^M} - (B^N - 1) i_{III}^N \right] i_{III\,I} \, i_0 \omega_0 = 0 \tag{56}$$

From this the transmission:

$$\frac{\omega_\infty}{\omega_0} = i_0 i_\infty \left[\frac{B^M}{B^N} i_{II}^M i_{II}^N - \frac{(B^M - 1) \dfrac{i_{II}^M \, i_{II}^N}{i_{III}^M} - (B^N - 1) i_{III}^N}{B^N} i_{III\,I} \right] \tag{57}$$

In a simple case, when

$$i_{II}^M = i_{II}^N = i_{III}^M = i_{III}^N = 1$$

and

$$i_0 = i_\infty = 1$$

the transmission:

$$\frac{\omega_\infty}{\omega_0} = \frac{B^M}{B^N} - \frac{B^M - B^N}{B^N} \, i_{III\,I} \tag{58}$$

resp. in case of closed clutch $(i_{III\,I} = 0)$

(59)

$$\frac{\omega_\infty}{\omega_0} = \frac{B^M}{B^N}$$

42. Kinematical conditions of the transfastened complex epicyclic gear

421. Kinematical gear transmission of the simple transfastened epicyclic gear at transfastening. At a transfastened simple epicyclic gear the number of the possible connecting elements is four (Fig. 32). The kinematical transmission of these: (49), (51), respectively:

$$i_{II} = \frac{\omega_x}{\omega_{II}} \text{ and } i_{\infty} = \frac{\omega_{\infty}}{\omega_x} \tag{60}$$

The angular speeds of the epicyclic gear's three shafts can be expressed from these:

From (49):

$$\omega_{I} = i_0 \omega_0 \tag{61}$$

From (60):

$$\omega_{II} = \frac{\omega_{\infty}}{i_{II}\, i_{\infty}} \tag{62}$$

From (60) and (51):

$$\omega_{III} = \frac{\omega_{\infty}}{i_{III}\, i_{\infty}} \tag{63}$$

Formulae (61), (62) and (63) substituted in the general kinematical (32) basic equation of the simple epicyclic gear we obtain:

$$B\, i_0 \omega_0 - \frac{\omega_{\infty}}{i_{II}\, i_{\infty}} - (B-1)\frac{\omega_{\infty}}{i_{III}\, i_{\infty}} = 0 \tag{64}$$

From this the transmission of the transfastened complex epicyclic gear can be expressed:

$$\frac{\omega_{\infty}}{\omega_0} = i_0 i_{\infty} \frac{B}{\frac{1}{i_{II}} + (B-1)\frac{1}{i_{III}}} \tag{65}$$

In a simple case (Fig. 29) when

$$i_0 = i_{\infty} = 1$$

and

$$i_{II} = 1$$

the transmission is

$$\frac{\omega_{\infty}}{\omega_0} = \frac{B}{1 + (B-1)\frac{1}{i_{III}}} \tag{66}$$

422. Kinematical gear transmission of the transfastened complex epicyclic gear. In case of transfastening (Fig. 39) shaft $\overline{III}_1$ of the complex epicyclic gear, after substitutions, the (45) basic equation is:

$$B^M i_{\overline{II}}^M i_{\overline{II}}^N i_0 \omega_0 - B^N \frac{\omega_\infty}{i_{\overline{II}}\, i_\infty} - \left[(B^M-1)\frac{i_{\overline{II}}^M i_{\overline{II}}^N}{i_{\overline{III}}^M} - (B^N-1)\, i_{\overline{III}}^N\right]\frac{\omega_\infty}{i_{\overline{III}}\, i_\infty} = 0 \qquad (67)$$

From this the transmission:

$$\frac{\omega_\infty}{\omega_0} = \frac{i_0\, i_\infty}{\dfrac{B^N}{B^M}\,\dfrac{1}{i_{\overline{II}}^M i_{\overline{II}}^N i_{\overline{II}}} + \left[\left(1-\dfrac{1}{B^M}\right)\dfrac{1}{i_{\overline{III}}^M} - \dfrac{(B^N-1)\, i_{\overline{III}}^N}{B^M\, i_{\overline{II}}^M i_{\overline{II}}^N}\right]\dfrac{1}{i_{\overline{III}}}} \qquad (68)$$

In a simple case (Fig. 34) when

$$i_{\overline{II}}^M = i_{\overline{II}}^N = i_{\overline{III}}^M = i_{\overline{III}}^N = 1$$

$$i_0 = i_\infty = 1$$

$$i_{\overline{II}} = 1$$

the transmission is:

$$\frac{\omega_\infty}{\omega_0} = \frac{1}{\dfrac{B^N}{B^M} + \left(1-\dfrac{B^N}{B^M}\right)\dfrac{1}{i_{\overline{III}}}} \qquad (69)$$

43. Kinematical conditions of the crossfastened complex epicyclic gear

431. Kinematical transmission of the crossfastened straight complex epicyclic gear. The number of the applicable connecting elements at crossfastening is four in the inside, three in the outside (Fig. 40). The kinematical transmission of these:

$$i_{\overline{II}}^M = \frac{\omega_{\overline{III}2}}{\omega_{\overline{II}}^M} \qquad (70)$$

$$i_{\overline{II}}^N = \frac{\omega_{\overline{II}}^N}{\omega_{\overline{III}2}} \qquad (71)$$

$$i_{\overline{III}}^M = \frac{\omega_{\overline{III}1}}{\omega_{\overline{III}}^M} \qquad (72)$$

$$i_{\overline{III}}^{N} = \frac{\omega_{\overline{III}}^{N}}{\omega_{\overline{III}1}} \tag{73}$$

respectively

$$i_{o} = \frac{\omega_{I}}{\omega_{o}} \tag{74}$$

$$i_{\infty} = \frac{\omega_{\infty}}{\omega_{\overline{II}}} \tag{75}$$

$$i_{\overline{III}} = \frac{\omega_{\overline{III}2}}{\omega_{\overline{III}1}} \tag{76}$$

Substituting formulae (74), (75), (76) into equation (45):

$$B^{M} i_{\overline{II}}^{M} i_{\overline{II}}^{N} i_{o}\omega_{o} - B^{N} \frac{\omega_{\infty}}{i_{\infty}} - \left[(B^{M}-1)\frac{i_{\overline{II}}^{M} i_{\overline{II}}^{N}}{i_{\overline{III}}^{M}} - (B^{N}-1) i_{\overline{III}}^{N}\right] \frac{1}{i_{\overline{III}}} \omega_{\overline{III}2} = 0 \tag{77}$$

From this we express $\omega_{\overline{III}2}$ and substitute it into equation (46). After ordination:

$$\frac{\omega_{\infty}}{\omega_{o}} = i_{o} i_{\infty} \frac{\left(1-\frac{1}{B^{N}}\right) i_{\overline{III}}^{N} + \frac{1}{B^{N}} i_{\overline{II}}^{N} i_{\overline{III}}}{\left(1-\frac{1}{B^{M}}\right) \frac{1}{i_{\overline{III}}^{M}} + \frac{1}{B^{M}} \frac{1}{i_{\overline{II}}^{M}} i_{\overline{III}}} \tag{78}$$

In a simple case (Fig. 35) when

$$i_{\overline{II}}^{M} = i_{\overline{II}}^{N} = i_{\overline{III}}^{M} = i_{\overline{III}}^{N} = 1$$

and

$$i_{o} = i_{\infty} = 1$$

the transmission is:

$$\frac{\omega_{\infty}}{\omega_{o}} = \frac{\left(1-\frac{1}{B^{N}}\right) + \frac{1}{B^{N}} i_{\overline{III}}}{1-\frac{1}{B^{M}} + \frac{1}{B^{M}} i_{\overline{III}}} \tag{79}$$

<u>432. Kinematical transmission of the crossfastened inverted complex epicyclic gear.</u> The number of the applicable connecting elements is identical to the case of the straight epicyclic gear. The transmission of the outside connecting elements is in this case:

$$i_{III} = \frac{\omega_{II}}{\omega_{I}} \tag{80}$$

$$i_{o} = \frac{\omega_{III_1}}{\omega_{o}} \tag{81}$$

$$i_{\infty} = \frac{\omega_{\infty}}{\omega_{III_2}} \tag{82}$$

These substituted into equations (45) and (46), after ordination we obtain:

$$\frac{\omega_{\infty}}{\omega_{o}} = i_{o}\, i_{\infty} \frac{B^{M}(B^{N}-1)\, i_{III}^{M} i_{III}^{N} \frac{1}{i_{III}} - B^{N}(B^{M}-1)}{B^{M} i_{I}^{M} i_{II}^{N} \frac{1}{i_{III}} - B^{N}} \; \frac{(B^{M}-1)\frac{i_{II}^{M} \; i_{I}^{N}}{i_{III}^{M} \; i_{III}^{N}} - (B^{N}-1)}{(B^{N}-1)\frac{i_{III}^{M} \; i_{III}^{N}}{i_{I}^{M} \; i_{I}^{N}} - (B^{M}-1)} \; \frac{i_{III}^{N}}{i_{I}^{N}} \tag{83}$$

In a simple case (Fig. 36) when

$$i_{I}^{M} = i_{II}^{N} = i_{III}^{M} = i_{III}^{N} = 1$$

and

$$i_{o} = i_{\infty} = 1$$

the transmission is:

$$\frac{\omega_{\infty}}{\omega_{o}} = -\frac{B^{M}(B^{N}-1)\frac{1}{i_{III}} - B^{N}(B^{M}-1)}{B^{M}\frac{1}{i_{III}} - B^{N}} \tag{84}$$

5. TORQUE CONDITIONS OF COMPLEX EPICYCLIC GEARS

51. Torque transmission of the simply fastened epicyclic gear

Outside torque conditions of epicyclic gears are characterized by the torque transmission.

At the simply fastened simple epicyclic gear (Fig. 31):

$$M_{\infty} = k_{\infty} M_{I} \tag{85}$$

$$M_{o} = \frac{M_{I}}{k_{o}} \tag{86}$$

Taking in consideration the proportion (33), we can write:

$$\frac{M_{\infty}}{M_{o}} = k_{o} k_{\infty}\left(-\frac{1}{B}\right) \tag{87}$$

In a simple case (Fig. 28):

$$\frac{M_\infty}{M_0} = -\frac{1}{B} \tag{88}$$

In case of a complex epicyclic gear, taking in consideration proportion (47) as regards shafts $\underline{I}-\underline{II}-\underline{III}_1$ (Fig. 38):

$$\frac{M_\infty}{M_0} = k_0 k_\infty \left[k^M \left(\frac{1}{B^M} - 1 \right) - \frac{k_{\underline{I}}^M k_{\underline{I}}^N}{k_{\underline{III}}^N} \; \frac{1 - B^N}{B^M} - 1 \right] \tag{89}$$

resp. the proportion (41) (Fig. 33):

$$\frac{M_\infty}{M_0} = -\frac{B^N}{B^M} \tag{90}$$

52. Torque transmission of the transfastened epicyclic gear

Let us write the torque-equilibrium for the connection-point of shaft X:

$$k_{\underline{III}} M_{\underline{III}} + k_{\underline{II}} M_{\underline{II}} + \frac{M_\infty}{k_\infty} = 0 \tag{91}$$

Taking in consideration relations (85), (86) and (33) we can write:

$$M_{\underline{II}} = -\frac{k_0}{B} M_0 \tag{92}$$

$$M_{\underline{III}} = \left(\frac{1}{B} - 1 \right) k_0 M_0 \tag{93}$$

Substituting expressions (92) and (93) into equation (91), after ordination we obtain for the torque-modification:

$$\frac{M_\infty}{M_0} = k_0 k_\infty \frac{(B-1) k_{\underline{III}} - k_{\underline{II}}}{B} \tag{94}$$

In a simple case (Fig. 29) the torque-modification is:

$$\frac{M_\infty}{M_0} = \left(1 - \frac{1}{B} \right) k_{\underline{III}} - \frac{1}{B} \tag{95}$$

In case of the complex epicyclic gear (Fig. 39), taking in consideration in the above deduction instead of the proportion (33) the (47), the torque modification for the shafts $\underline{I}-\underline{II}-\underline{III}_1$ will be in the general case:

$$\frac{M_\infty}{M_0} = k_0 k_\infty \left\{ k_{\underline{III}} \left[k_{\underline{III}}^M \left(\frac{1}{B^M} - 1 \right) - \frac{k_{\underline{I}}^M k_{\underline{I}}^N}{k_{\underline{III}}^N} \; \frac{1-B^N}{B^M} \right] - k_{\underline{II}} \left[\left(1 - \frac{1}{B^M} \right) + \frac{k_{\underline{I}}^M k_{\underline{I}}^N}{k_{\underline{III}}^N} \; \frac{1-B^N}{B^M} - 1 \right] \right\} \tag{96}$$

In a simple case (Fig. 34):

$$\frac{M_\infty}{M_0}=\left(1-\frac{B^N}{B^M}\right)k_{III}-\frac{B^N}{B^M} \tag{97}$$

53. Torque transmission of the crossfastened epicyclic gear

Let us write the torque-equilibrium at the crossfastened straight complex epicyclic gear (Fig. 40) for the connection-point of the shafts of connecting element i_{III}:

$$k_{III}^M M_{III}^M+\frac{1}{k^N} M_{III}^N+M_{III_1}=0 \tag{98}$$

$$k_{II}^M M_{II}^M+\frac{1}{k^N} M_{II}^N+M_{III_2}=0 \tag{99}$$

Since

$$M_{III}^M=\left(\frac{1}{B^M}-1\right)M_I^M=k_0\left(\frac{1}{B^M}-1\right)M_0 \tag{100}$$

$$M_{III}^N=\left(\frac{1}{B^N}-1\right)M_I^N=\frac{1}{k_\infty}\left(\frac{1}{B^N}-1\right)M_\infty \tag{101}$$

$$M_{II}^M=\left(-\frac{1}{B^M}\right)M_I^M=k_0\left(-\frac{1}{B^M}\right)M_0 \tag{102}$$

$$M_{II}^N=\left(-\frac{1}{B^N}\right)M_I^N=\frac{1}{k_\infty}\left(-\frac{1}{B^N}\right)M_\infty \tag{103}$$

and

$$M_{III_1}=\frac{1}{k_{III}}M_{III_2} \tag{104}$$

therefore the equations (98) and (99) can be written in the following form too:

$$k_0 k_{III}^M\left(\frac{1}{B^M}-1\right)M_0+\frac{1}{K_{III}^N k_\infty}\left(\frac{1}{B^N}-1\right)M_\infty+\frac{1}{k_{III}}M_{III_2}=0 \tag{105}$$

$$k_0 k_{II}^M\left(-\frac{1}{B^M}\right)M_0+\frac{1}{k_{II}^N k_\infty}\left(-\frac{1}{B^M}\right)M_\infty+M_{III_2}=0 \tag{106}$$

Let us multiplicate the first equation of the equation-pair by k_{III} and substract from it the second equation, then we obtain after ordination:

$$\frac{M_\infty}{M_0}=-k_0 k_\infty\frac{\frac{1}{B^M}k_{II}^M+\left(\frac{1}{B^M}-1\right)k_{III}^M k_{III}}{\frac{1}{B^N}\frac{1}{k_{II}^N}+\left(\frac{1}{B^N}-1\right)\frac{1}{k_{III}^N}k_{III}} \tag{107}$$

In a simple case (Fig. 35):

$$\frac{M_\infty}{M_0} = \frac{\frac{1}{B^M} + \left(\frac{1}{B^M} - 1\right) k_{\mathrm{III}}}{\frac{1}{B^N} + \left(\frac{1}{B^N} - 1\right) k_{\mathrm{II}}} \tag{108}$$

By a similar deduction we obtain also for the inverted complex epicyclic gear (Fig. 41):

$$\frac{M_\infty}{M_0} = k_0 k_\infty \frac{B^M(B^N-1)\frac{k_{\mathrm{III}}}{k^M_{\mathrm{II}}\, k^N_{\mathrm{II}}} + B^N(B^M-1)}{B^M \frac{k_{\mathrm{III}}}{k^M_{\mathrm{II}}\, k^N_{\mathrm{II}}} + B^N} \quad \frac{(B^M-1)\frac{k^M_{\mathrm{III}}\, k^N_{\mathrm{III}}}{k^M_{\mathrm{II}}\, k^N_{\mathrm{II}}} - (B^N-1)}{(B^N-1)\frac{k^M_{\mathrm{I}}\, k^N_{\mathrm{I}}}{k^M_{\mathrm{III}}\, k^N_{\mathrm{III}}}} \quad \frac{k^M_{\mathrm{I}}}{k^N_{\mathrm{III}}} \tag{109}$$

resp. for the simple case (Fig. 36):

$$\frac{M_\infty}{M_0} = -\frac{B^M(B^N-1)k_{\mathrm{III}} - B^N(B^M-1)}{B^M k_{\mathrm{III}} - B^N} \tag{110}$$

It can be checked that all the expressions obtained for torque transmission can also be directly obtained from the formula of the corresponding kinematical transmission by

$$i = -\frac{1}{k} \tag{111}$$

resp. by

$$\frac{\omega_\infty}{\omega_0} = -\frac{M_0}{M_\infty} \tag{112}$$

substitution.

6. PERFORMANCE CONDITIONS OF COMPLEX EPICYCLIC GEARS

61. Performance-flow of the simply fastened epicyclic gear

We are interested in the performance conditions of complex epicyclic gears from the point of view of distribution of performance according to the shafts, resp. of the flow of the performance. For starting our examinations we will use the performance-proportions.

At the simply fastened epicyclic gear the performance flows in one direction, since on the base no performance can fall. In case of straight drive the performance comes in on the O shaft, goes out on the ∞ shaft, at inverted drive the ∞ shaft is the in-coming and the O shaft the out-going shaft. If the simple fastening is made by such connecting element in which losses occur (e.g. by hydrodinamic clutch) then the incoming and outgoing performances obviously will not be identical, in spite of that, that the epicyclic gear itself is regarded as free of losses.

62. Performance-flow of the transfastened epicyclic gear

621. Performance-flow of the transfastened simple epicyclic gear. At transfastened epicyclic gears (Fig. 32) in general performance-ramification occurs, consequently examination of the distribution and direction of the performance-flow is an important task.

We get a characteristic view of the performance-flow conditions by comparing the signs of the performances flowing on the individual shafts.

From proportion (34):

$$\frac{P_{II}}{P_{I}} = \left(-\frac{1}{B}\right)\frac{\omega_{II}}{\omega_{I}} \tag{113}$$

We express the angular speed by the angular speeds of shafts ∞ , resp. O, using formulae (62) and (65):

$$\omega_{II} = \frac{\omega_{\infty}}{i_{II}\ i_{\infty}} = \frac{\omega_o}{i_{II}\ i_{\infty}} = \frac{\omega_{\infty}}{\omega_o} = \frac{i_o}{i_{II}}\ \frac{B}{\frac{1}{i_{II}}+\frac{B-1}{i_{III}}} \tag{114}$$

By substituting formulae (61) and (114) into expression (113) we obtain:

$$\frac{P_{II}}{P_{I}} = \frac{1}{\frac{i_{II}}{i_{III}}(1-B)-1} \tag{115}$$

The signs of the performances on shafts I and II are identical, i.e.

$$\frac{P_{II}}{P_{I}} > 0 \tag{116}$$

if

$$\frac{i_{II}}{i_{III}}(1-B) > 1 \tag{117}$$

Again from the proportion (34):

$$\frac{P_{III}}{P_{I}} = \left(\frac{1}{B}-1\right)\frac{\omega_{III}}{\omega_{I}} \tag{118}$$

By using formulae (58) and (59):

$$\omega_{III} = \frac{\omega_{\infty}}{i_{III}\ i_{\infty}} = \frac{\omega_o}{i_{III}\ i_{\infty}}\ \frac{\omega_{\infty}}{\omega_o} = i_o\frac{B}{\frac{i_{III}}{i_{II}}+B-1}\,\omega_o \tag{119}$$

Formulae (61) and (119) substituted into expression (118) we obtain:

$$\frac{P_{III}}{P_{I}} = \frac{1}{\frac{i_{III}}{i_{II}}\ \frac{1}{1-B}-1} \tag{120}$$

The signs of the performances on shafts I and III are identical, i.e.

$$\frac{P_{III}}{P_I} > 0 \tag{121}$$

if

$$\frac{i_{III}}{i_{II}} \frac{1}{1-B} > 1 \tag{122}$$

resp. by putting it in a form similar to that of expression (117):

$$\frac{1}{\frac{i_{II}}{i_{III}}(1-B)} > 1 \tag{123}$$

On base of inequalities (117) and (123) all possible cases of performance--ramification can be determined.

If (117) is valid then (123) can not be valid i.e. in this case the sign of P_{III} differs from those of P_I and of P_{II} as well. This means that in case of straight drive shafts I and II are in-coming shafts, while shaft III is out-going i.e. trough shaft II performance flows back into the epicyclic gear (performance-circulation). In Fig. 43 the direction of performance-flow is shown by arrows. Since values i_0 and i_∞ have no influence on the direction of the performance-flow, for the sake of simplicity these were not indicated. In case of reversed drive there also is performance-circulation, here, however, the epicyclic gear "sends back" the circulating performance (Fig. 44).

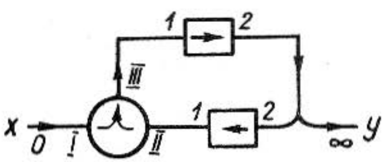


Fig. 43

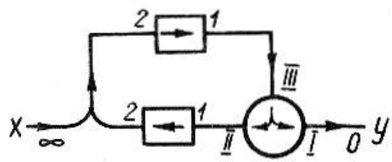


Fig. 44.

If (117) is not valid then two possibilities are met with. Inasmuch

$$0 < \frac{i_{II}}{i_{III}}(1-B) < 1 \tag{124}$$

then (123) is valid and in such case the sign of P_{II} differs from those of the other two performances. We have performance circulation also in this case but with an opposed direction of rotation (Figs. 45 and 46).

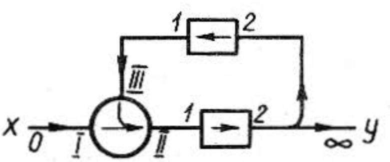


Fig. 45.

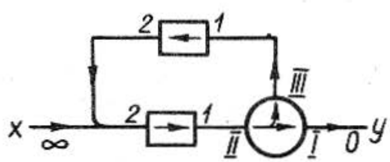


Fig. 46.

If

$$\frac{i_{II}}{i_{III}}(1-B) < 0 \tag{125}$$

then neither (117) nor (123) are valid and in this case the sign of P_I differs from those of the other two performances. We have no performance-circulation now, only a simple performance-ramification resp. the performance proceeds parallel. (Figs. 47 and 48).

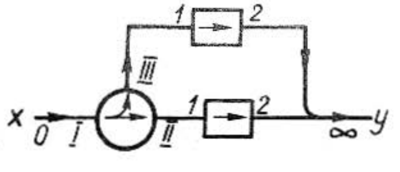


Fig. 47.

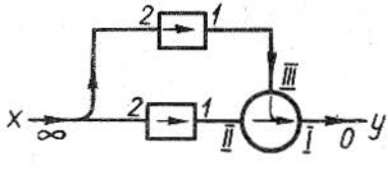


Fig. 48.

By examining the inequalities (117), (124) and (125) the direction of the performance-process on the individual connecting elements may simply be determined and on base of this the correct manner of fastening of the relevant connecting element can be determined too(e.g. it is reasonable to use a hydraulic torque converter only in the case when the performance proceeds from the pump-impeller towards the turbine wheel). In the last column of the Table of the following paragraph also these three cases are indicated.

The absolute extent of the performance flowing on the individual shafts can be determined by means of formulae (115) and (120).

From the above otherwise it also can be seen that at the transfastened epicyclic gear we can not unambigously speak of double driving-in or double driving-out, on base of the location of the epicyclic gear.

622. Performance-flow ot the transfastened complex epicyclic gear. First the performance-flow outside of the epicyclic gear will be examined (Fig. 39). The deduction will be dispensed with since it is totally analogous to the deduction of formulae (115) and (120), only instead of proportion (34) the proportion (47) must be used as starting point and instead of formula (65) the formula (68) has to be used.

The result is:

$$\frac{P_{II}}{P_I} = \frac{k^M_{III}(1-B^M) - \frac{k^M_{I}\, k^N_{II}}{k^N_{II}}(1-B^N) - B^M}{\frac{1}{i^M_{II}\; i^N_{I}} B^N + \frac{i_{II}}{i_{III}}\left[\frac{1}{i^M_{III}}(B^M-1) - \frac{i^N_{III}}{i^M_{II}\; i^N_{II}}(B^N-1)\right]} \tag{126}$$

resp.

$$\frac{P_{III}}{P_I} = \frac{k^M_{III}(B^M-1) - \frac{k^M_{II}\, k^N_{II}}{k^N_{III}}(B^N-1)}{\frac{i_{III}}{i^M_{II}\; i^N_{II}\; i_{II}} B^N + \frac{1}{i^M_{III}}(B^M-1) - \frac{i^N_{III}}{i^M_{II}\; i^N_{II}}(B^N-1)} \tag{127}$$

In a simple case (Fig. 34):

$$\frac{P_{II}}{P_I} = \frac{1}{\frac{1}{i_{III}}\left(1-\frac{B^M}{B^N}\right)-1} \tag{128}$$

resp.

$$\frac{P_{III1}}{P_I} = \frac{1}{\frac{1}{\frac{1}{i_{III}}\left(1-\frac{B^{N'}}{B^N}\right)}-1} \tag{129}$$

As a matter of course the relation (38) is valid also in this case and taking this in consideration those spoken of in connection with the inequalities (117) and (123) can be applied according to the sense.

At the complex epicyclic gear, however, also the examination of the performance between the two simple epicyclic gears i.e. of that within the complex epicyclic gear may be needed. Within the complex epicyclic gear four kinds of performance-flow are possible at all three performance-conditions of the transfastened epicyclic gear. In the horizontal rows of Table 3 are arranged the cases identical from viewpoint of inequality and in each row four varieties can be seen for the performance-flow within the complex epicyclic gear.

For the examination of the inside performance-flow the performance-proportions $\frac{P_{II}^N}{P_{III}^N}$ and $\frac{P_{II}^M}{P_{III}^M}$ give information. If both quotients have negative sense at the same time, i.e.

$$\frac{P_{II}^M}{P_{III}^M} < 0 \tag{130}$$

and

$$\frac{P_{II}^N}{P_{III}^N} < 0 \tag{131}$$

then the performance circulates in the complex epicyclic gear (first two columns). The direction of the circulation is shown by the sign of quotient $\frac{P_{II}^M}{P_I}$.

If

$$\frac{P_{II}^M}{P_I} > 0$$

then the performance-circulation is formed according to the figures in the first column, in the contrary case the figures in the second column are relevant.

If of the quotients $\frac{P_{II}^M}{P_{III}^M}$ and $\frac{P_{II}^N}{P_{III}^N}$ one or both are positive, we obtain unambigously the direction of the performance-flow, that is, if the value of

$\frac{P_{II}^{N}}{P_{III}^{N}}$ exceeds O then the fourth column, finally, if both quotients are positive then the Figs. of the fifth column show the direction. In Table 3 the criterions pertaining to each 12 cases are indicated.

The direction of the performance-flow is marked for the case of straight drive. In case of reversed drive each arrow must be drawn in the opposite direction.

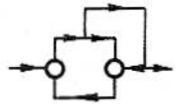
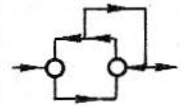
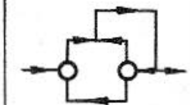
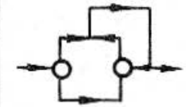
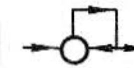
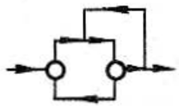
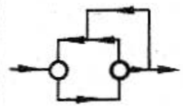
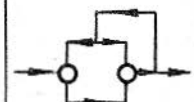
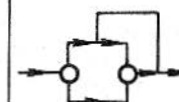
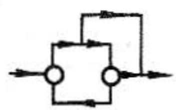
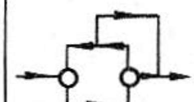
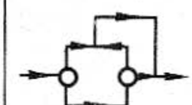
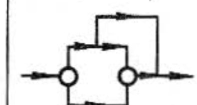
Values of the individual quotients can easily be calculated, here we give the formulae only for the simple case (Fig. 34), dispensing with the deducions:

$$\frac{P_{II}^{M}}{P_{III}^{M}} = \frac{B^{N}(i_{III}-1)+1}{B^{M}-1} \tag{133}$$

$$\frac{P_{II}^{N}}{P_{III}^{N}} = \frac{i_{III}}{1-\frac{1}{B^{N}}} - 1 \tag{134}$$

and it is needed at the first two columns:

$$\frac{P_{II}^{M}}{P_{I}^{M}} = \frac{1}{(B^{N}-B^{M})\frac{1}{i_{III}} - B^{N}} \tag{135}$$

	$B_1 = \frac{B^M}{B^N}$					B
$\frac{1-B}{i_{III}} > 1$					—	
$0 < \frac{1-B}{i_{III}} < 1$				—		
$\frac{1-B}{i_{III}} < 0$			—			
P_{II}^M / P_{III}^M	< 0		< 0	> 0	> 0	
P_{II}^N / P_{III}^N	< 0		> 0	< 0	> 0	
P_{III}^M / P_I	> 0	< 0				

63. Performance-flow of the cross fastened epicyclic gear

At the cross fastened epicyclic gear only the sign of the performance proceeding through i_{III} connecting element has to be examined, since here only two cases are possible: the performance proceeds from shaft III_1 towards shaft III_2 or the reverse of this.

The methods of the examination are similar both at the straight and the inverted complex epicyclic gears. In the following we deduct only the formulae of the straight epicyclic gear, those of the inverted ones can be obtained in an analogous way.

From proportion (47)

$$\frac{P_{\mathrm{III}1}}{P_{\mathrm{I}}}=\left[k_{\mathrm{III}}^{M}\left(1-\frac{1}{B^{M}}\right)-\frac{k_{\mathrm{II}}^{M}\,k_{\mathrm{II}}^{N}}{k_{\mathrm{III}}^{N}}\;\frac{B^{N}-1}{B^{M}}\right]\frac{\omega_{\mathrm{III}1}}{\omega_{\mathrm{I}}} \tag{136}$$

From the general kinematical basic equation of B^M complex epicyclic gear

$$\omega_{\mathrm{III}}^{M}=\frac{B^{M}\omega_{\mathrm{I}}^{M}-\omega_{\mathrm{II}}^{M}}{B^{M}-1} \tag{137}$$

Taken in consideration that

$$\omega_{\mathrm{III}}^{M}=\frac{\omega_{\mathrm{III}1}}{i_{\mathrm{III}}^{M}} \tag{138}$$

the

$$\omega_{\mathrm{II}}^{M}=\frac{i_{\mathrm{III}}}{i_{\mathrm{II}}^{M}}\;\omega_{\mathrm{III}1} \tag{139}$$

from the equation can be expressed:

$$\omega_{\mathrm{III}1}=\frac{\omega_{\mathrm{I}}}{\left(1-\frac{1}{B^{M}}\right)\frac{1}{i_{\mathrm{III}}^{M}}+\frac{1}{B^{M}}\;\frac{1}{i_{\mathrm{II}}^{M}}\,i_{\mathrm{III}}} \tag{140}$$

If this is substituted into equation (136) we obtain for the performance flowing on shaft III_1

$$\frac{P_{\mathrm{III}1}}{P_{\mathrm{I}}}=\frac{(B^{M}-1)k_{\mathrm{III}}^{M}-\frac{k_{\mathrm{II}}^{M}\,k_{\mathrm{II}}^{N}}{k_{\mathrm{III}}^{N}}\,(B^{N}-1)}{(B^{M}-1)\,\frac{1}{i_{\mathrm{III}}^{M}}+\frac{i_{\mathrm{III}}}{i_{\mathrm{II}}^{M}}} \tag{141}$$

In a simple case (Fig. 35):

$$\frac{P_{\mathrm{III}1}}{P_{\mathrm{I}}}=\frac{B^{N}-1}{i_{\mathrm{III}}+B^{M}-1} \tag{142}$$

Here also the examining of the performance-flow between the two simple epicyclic gears i.e. of that within the complex epicyclic gear can be needed. The method is similar to that used at the transfastened epicyclic gear.

The two quotients:

$$\frac{P_{II}^{M}}{P_{III}^{M}} = \frac{1}{B^{M}-1}\, i_{III} \tag{143}$$

$$\frac{P_{II}^{N}}{P_{III}^{N}} = \frac{1}{B^{N}-1}\, i_{III} \tag{144}$$

If the values of both quotients are negative then for the determination of the circulation's direction also the sign of the

$$\frac{P_{II}^{M}}{P_{I}^{M}} = \frac{1}{\frac{1-B^{M}}{i_{III}}-1} \tag{145}$$

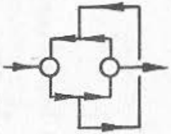
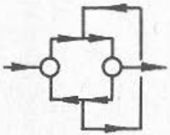
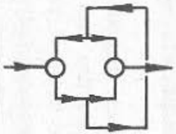
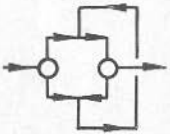
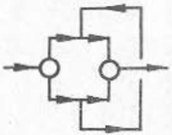
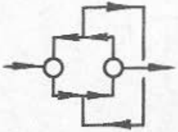
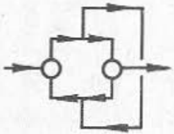
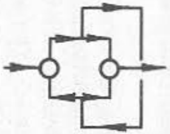
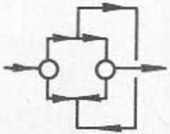
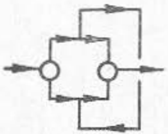
quotient is needed. The possible ten directions of performance-flow in case of straight drive is shown in Table 4, where the ciretions pertaining to the individual cases are indicated as well.

By means of the deducted relations the kinematical, torque and performance conditions of any epicyclic gear structure can simply and rapidly be examined. The formulae given refer to straight drive, in case of reversed drive the reciprocals of the ratios will be taken. The kinematical transmission e.g. at a straight driven transfastened epicyclic gear (socalled prefastened epicyclic gear) is:

$$i_{\infty 0} = \frac{\omega_{\infty}}{\omega_{0}} = \frac{B}{1+(B-1)\,\frac{1}{i_{III}}}$$

The same in case of reversed drive i.e. at backfastened epicyclic gear:

$$i_{0\infty} = \frac{\omega_{0}}{\omega_{\infty}} = \frac{1}{B} + \left(1+\frac{1}{B}\right)\frac{1}{i_{III}} \tag{146}$$

$\frac{P_{III1}}{P_I} = \frac{B^N - 1}{i_{III} + B^M - 1} > 0$					
$\frac{P_{III1}}{P_I} = \frac{B^N - 1}{i_{III} + B^M - 1} < 0$					
$\frac{P_{II}^M}{P_{III}^M} = \frac{i_{III}}{B^M - 1}$	<0		<0	>0	>0
$\frac{P_{II}^N}{P_{III}^N} = \frac{i_{III}}{B^N - 1}$	<0		>0	<0	>0
$\frac{P_{II}^M}{P_I} = \frac{1}{\frac{1 - B^M}{i_{III}} - 1}$	<0	>0			

III. MECHANICAL CHANGE-SPEED GEARS WITH EPICYCLIC GEARS (98)

1. THE COMPLEX EPICYCLIC GEARS AS MECHANICAL CHANGE-SPEED STAGES

We are dealing with a mechanical change-speed gear whenever all its connecting elements are mechanical power transmission structures, in the first place a clutch or some kind of one degree of freedom geared drive.

In the following will be examined in what kinds of concrete forms the complex epicyclic gears can appear in mechanical change-speed gears and it will be illustrated on examples how they may be examined by means of the methods resp. formulae described in the foregoing paragraphs.

The mechanical clutch - as all clutches - is characterized by the fact that it does not modify torque ($k = -1$). From viewpoint of the taken resp. transmitted torque's extent the clutches from our point of view can be numbered to two groups: there are controlled and automatic clutches.

At controlled clutches the transmitted torque is regulated according to some outside law. The torque taken is regulated e.g. at frictional clutches (brakes) by the variation of the force compressing the chafing bodies, at hydrodinamical clutches by the variation of the liquid's quantity and at electric clutches by that of the excitation or load.

At automatic clutches the transmitted torque depends on the service-conditions of the clutch itself; at a hydrodinamical clutch with constant charge e.g. on the rpm of the turbine wheel. So at automatic clutches the torque characteristics may be taken as a given one. Let us take in our example the controlled clutch. In practice mainly two service-conditions occour: open (released) and closed conditions. Sliding resp. letting to slide can be taken as a transitory state. (Sometimes also slided controlled clutch occurs e.g. at steering motor vehicles with chain track.)

In Fig. 49 a simply fastened epicyclic gear is seen where the connecting element is a frictional clutch (brake). The clutch being in released state, the III element of the epicyclic gear is kinematically undetermined, therefore then the epicyclic gear runs freely, it can not take part in the power transmission.

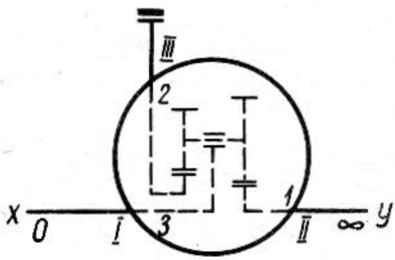


Fig. 49.

In case of closed clutch at straight drive the kinematical modification according to formula (55) is:

$$i = i_{\infty 0} = \frac{\omega_\infty}{\omega_0} = B$$

Let $b = 0,8$. Since fastening of the elementary epicyclic gear occurred according to the second variation (Table 1):

$$i = B = \frac{b-1}{b} = \frac{0,8-1}{0,8} = -0,25$$

The torque modification is according to formula (88):

$$k_{\infty 0} = \frac{M_\infty}{M_0} = -\frac{1}{B} = 4$$

The simply fastened epicyclic gear so in case of closed clutch (brake) is transformed to a one degree of freedom mechanical power transmission structure with constant transmission (to change-speed gear, resp. torque converter, reductor, etc.).

We may use a clutch also for transfastening the epicyclic gear. At the transfastened epicyclic gear seen in Fig. 50 in case of closed clutch $i_{III} = 1$ i.e. the transmission of the epicyclic gear according to formula (66) is:

$$i = i_{\infty 0} = 1$$

that is the epicyclic gear is transformed to a rigidly together-clamping body (shortclosed epicyclic gear).

If as connecting element a mechanical structure with constant transmission (change speed gear resp. torque converter, reductor) is applied then it is valid:

$$i = -\frac{1}{k}$$

inasmuch we do not take also here the frictional losses in cosideration.

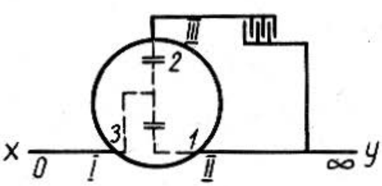


Fig. 50.

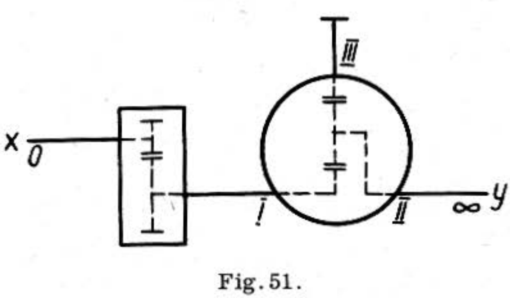


Fig. 51.

The mechanical structure with constant transmission has significance at a simply fastened epicyclic gear only then, when it is series connected with the former (Fig. 51).

We already mentioned above that the simply fastened epicyclic gear in case of closed clutch (brake) is transformed to a one degree of freedom mechanical structure with constant transmission (Fig. 49), therefore it can be used at an other epicyclic gear as a connecting element with such character. At the structure shown in Fig. 52 (1 stage of the Cotal change speed gear) two simply fastened epicyclic gears are series connected; theoretically either of them can be the connecting element of the other.

Fig. 53 shows an epicyclic gear transfastened by a geared structure of i_{III} transmission, where also a connecting element of i_∞ transmission can be seen which is solved also by geared wheels [1]

The block scheme of the epicyclic gear is presented in Fig. 54.

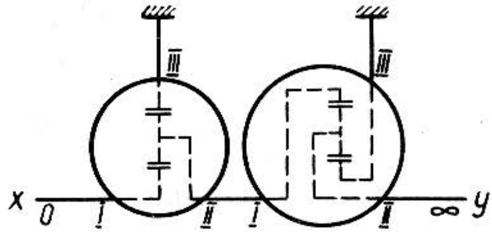


Fig. 52.

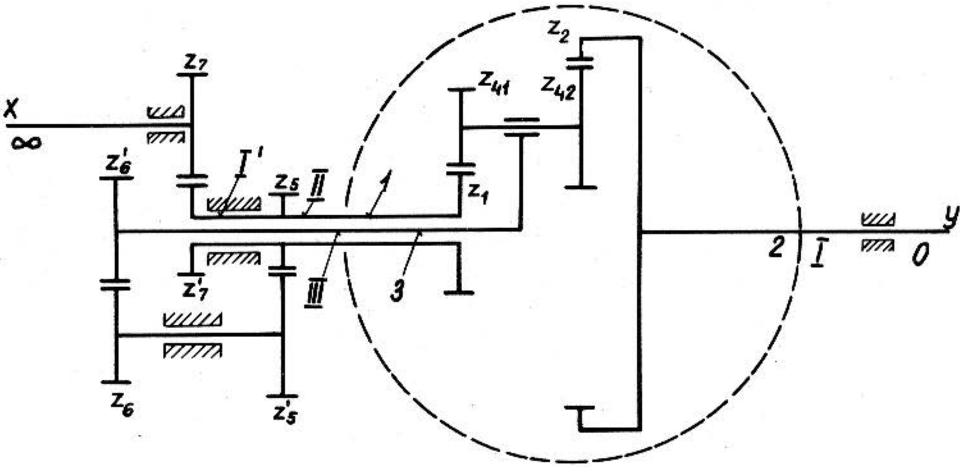


Fig. 53.

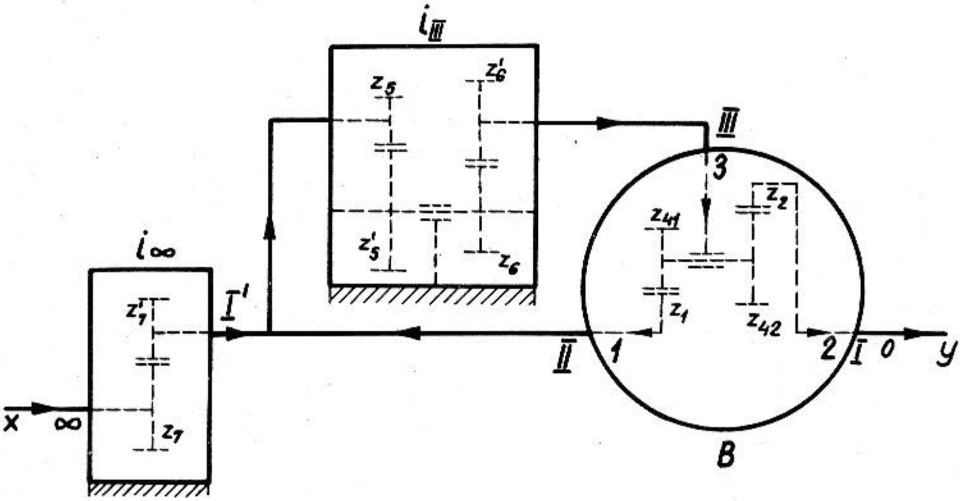


Fig. 54.

Let the constant transmission be:

$$i = u = -\frac{z_7}{z_7'} = -\frac{60}{40} = -1{,}5$$

$$i_{\mathrm{III}} = u_{\mathrm{III}} = \frac{z_6'}{z_6} \quad \frac{z_5'}{z_5} = \frac{30{,}60}{50{,}20} = 1{,}82$$

The basic transmission of the simple epicyclic gear is:

$$b = \frac{z_1}{z_{41}} \frac{z_{42}}{(-z_2)} = -\frac{40{,}50}{30{,}120} = -0{,}555$$

The binding-in corresponds to the fourth variation of Table 1, therefore the general basic transmission is:

$$B = \frac{1}{b} = -1{,}8$$

The complete transmission of the epicyclic gear in case of reversed drive is according to formula (65) taking in consideration that $i_o = 1$ and $i_{\mathrm{II}} = 1$:

$$i = \frac{\omega_o}{\omega_\infty} = i_{o\infty} = \frac{1+(B-1)\frac{1}{i_{\mathrm{III}}}}{B} = \frac{1+(-1{,}8-1)\frac{1}{1{,}82}}{(-1{,}5)(-1{,}8)} = -0{,}2$$

It can be seen that the epicyclic gear transfastened by the mechanical connecting element is transformed to a one degree of freedom mechanical structure with constant transmission. For the calculation of the torque modification use of the formula (94) is needless since we are dealing with a pure mechanical structure, when

$$k = -\frac{1}{i}$$

therefore

$$k_{o\infty} = 5$$

Let us calculate for the examination of the performance flow the value of expression

$$\frac{i_{\mathrm{II}}}{i_{\mathrm{III}}} (1-B)$$

which is

$$\frac{1}{1{,}82}(1+1{,}8) = 1{,}54 > 1$$

therefore the inequality (117) is valid and in Fig. 54 the direction of the performance-flow (circulation) was indicated on the base of this.

From those told in connection with Fig. 49 follows that the connecting element of the transfastened epicyclic gear can be a simply fastened epicyclic gear too. The connecting element of the transfastened epicyclic gear M with reversed drive seen in Fig. 55 (back-fastened epicyclic gear) is the simply fastened epicyclic gear N (second stage of Wilson).

Let the geometrical dimensions of the two simple epicyclic gears be uniform:

$$b^M = b^N = \frac{D_1}{D_2} = -0{,}4$$

If we connect shaft I of the N simply fastened gear to the shaft III of the epicyclic gear M (we may connected also the shaft II) then the binding-in of both epicyclic gears occured according to an identical variation:

$$B^M = B^N = \frac{b-1}{b} = \frac{-0{,}4-1}{-0{,}4} = 3{,}5$$

The transmission of the epicyclic gear N figuring as connecting element, is according to formula (55):

$$i = \frac{\omega_\infty}{\omega_0} = B^N = 3{,}5 = i_{III}$$

After all these the transmission of the back-fastened epicyclic gear M is according to formulae (66) and (146):

$$i = i_{0\infty} = \frac{\omega_0}{\omega_\infty} = \frac{1+(B^M-1)\frac{1}{i_{III}}}{B^M} = \frac{1+(3{,}5-1)\frac{1}{3{,}5}}{3{,}5} = 0{,}487$$

Let us calculate for the examination of the performance-flow the value of the expression

$$\frac{i_{II}}{i_{III}}(1-B^M)$$

which is

$$\frac{1}{3{,}5}(1-3{,}5) = -0{,}715 < 0$$

therefore the inequality (125) is valid, and the performance-flow occurs according to Fig. 48. In this case so both epicyclic gears are of reversed drive. When formely not shaft I but II of epicyclic gear should have been connected to shaft III of epicyclic gear M, then it would be now of straight drive, but of course, together with this also the binding-in should have been changed so that the same result would be arrived at anyhow. Reverting to Fig. 55 it can be stated that an other epicyclic gear transfastened by a simply fastened epicyclic gear will be changed to a complex epicyclic gear.

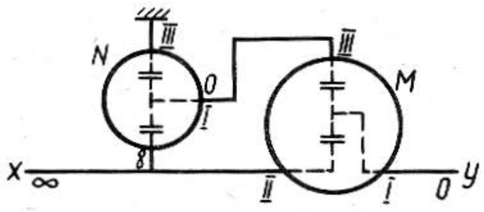


Fig. 55

Therefore, this special case of the transfastened epicyclic gear could also be dealt with as such a complex epicyclic gear at which the shaft to be fastened is changed with one of the shafts to be led out (Fig. 56). This way of dealing is, however, not expedient, since the unity of the system would be disrupted without obtaining any simplification at the same time. This possibility, otherwise, will be still referred to later on.

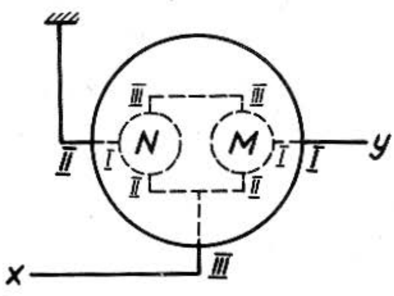


Fig. 56.

It was mentioned above, that the epicyclic gear transfastened by mechanical connecting element also will be transfastened to a one degree of freedom mechanical structure with constant transmission, therefore this also may be used in this character as connecting element in an other epicyclic gear.

We can meet with the aggregation of epicyclic gears e.g. in the third stage of the Wilson change speed gear. In Fig. 57. can be seen that the connecting element of transfastened epicyclic gear M is the transfastened epicyclic gear N in which the simply fastened epicyclic gear O is the connecting element.

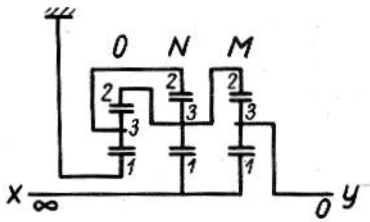


Fig. 57.

Here the I and II indices of the epicyclic gear N can be unambigously only those according to the Fig., since connecting element O has to connect unambigously shaft III with shaft II.

Values of b^M and b^N were already given at the second stage of the Wilson. The

$$b_C = -0{,}357$$

while the general basic transmissions

$$B^M = \frac{b-1}{b} = \frac{-0{,}4-1}{-0{,}4} = 3{,}5$$

$$B^N = \frac{b}{b-1} = \frac{-0{,}4}{-0{,}4-1} = 0{,}286$$

$$B^0 = 1-b = 1+0{,}357 = 1{,}357$$

It can be seen that now, as a matter of course, B^M and B^N are not equal. The transmission of the simply fastened epicyclic gear O is:

$$i^0 = \frac{\omega_{\infty}}{\omega_0} = B^0 = 1{,}357$$

which is at the same time the transmission of the connecting element of epicyclic gear N, i.e.

$$(i_{\text{III}})^N = 1{,}357$$

The transmission of the epicyclic gear N is according to the formula (66):

$$i = \frac{\omega_{\infty}}{\omega_0} = \frac{B^N}{1+(B^N-1)\frac{1}{i^N_{\text{III}}}} = \frac{0{,}286}{1+(0{,}286-1)\frac{1}{1{,}357}} = 0{,}6$$

which is at the same time the reciprocal of the transmission of the connecting element of epicyclic gear i.e.

$$(i_{\text{II}})^M = \frac{1}{0{,}6} = 1{,}67$$

After this the transmission of the change speed gear stage is:

$$\frac{\omega_0}{\omega_{\infty}} = \frac{1+(B^M-1)\frac{1}{i_{\text{III}}}}{B^M} = \frac{1+(3{,}5-1)\frac{1}{1{,}67}}{3{,}5} = 0{,}716$$

The performance-flow at epicyclic gear M, since

$$\frac{i_{\text{II}}}{i_{\text{III}}}(1-B^M) = \frac{1}{1{,}67}(1-3{,}5) = -1{,}5 < 0$$

is the one according to Fig. 48 and at epicyclic gear N since

$$\frac{i_{II}}{i_{III}}(1-B^{N}) = \frac{1}{1{,}357}(1-0{,}286) = 0{,}53$$

that according to Fig. 45. On base of this the direction of the performance-flow can be drawn also into Fig. 57.

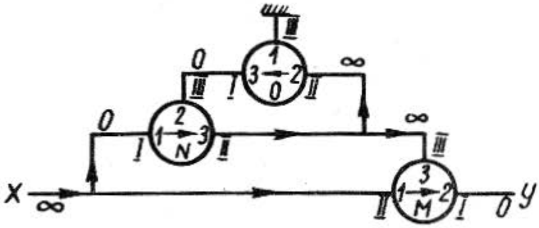
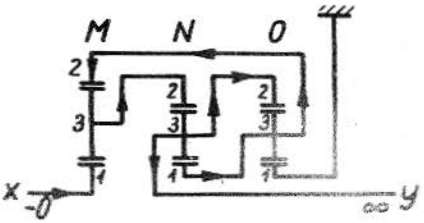


Fig. 57/II.

It can be seen that the drive of epicyclic gear M is reversed, while those of epicyclic gears N and O are straight. The stage is therefore in the reality a back-fastened epicyclic gear the connecting element of which is a third epicyclic gear prefastened by a simply fastened one.



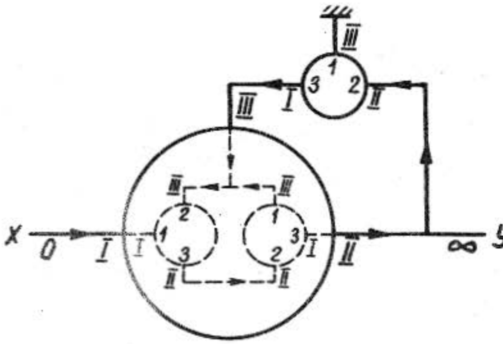


Fig. 58.

Fig. 59.

Let us present an example also for the complex epicyclic gear [7] (Fig. 58). The complex epicyclic gear consists of epicyclic gears M and N and this transfastened by the simply fastened epicyclic gear O. The block-scheme of the epicyclic gear is shown in Fig. 59.

The data of the elementary epicyclic gears are:

$$b^{M} = -0{,}336 \; ; \qquad B^{M} = \frac{b^{M}}{b^{M}-1} = \frac{-0{,}336}{-0{,}336-1} = 0{,}252$$

$$b^N = -0{,}614 \quad ; \qquad B^N = 1 - b^N = 1 + 0{,}614 \quad = 1{,}614$$

$$b^0 = -0{,}632 \quad ; \qquad B^0 = 1 - b^0 = 1 + 0{,}632 \quad = 1{,}632$$

The transmission of connecting element i_{III} is equal to the transmission of simply fastened epicyclic gear O, which is according to formula (69):

$$i = \frac{\omega_{\infty}}{\omega_0} = \frac{1}{\frac{B^N}{B^M} + \left(1 - \frac{B^N}{B^M}\right) \frac{1}{i_{III}}} = \frac{1}{\frac{1{,}614}{0{,}252} + \left(1 - \frac{1{,}614}{0{,}252}\right) \frac{1}{1{,}632}} = 0{,}33$$

For the examination of the performance-flow the expression (117) is, taking in consideration that $B = \frac{B^M}{B^N} = 0{,}16$ the following:

$$\frac{i_{II}}{i_{III}}(1 - B) = \frac{1}{1{,}632}(1 - 0{,}16) = 0{,}515 \lesseqgtr {}^{1}_{0}$$

Further performance proportions:

$$\frac{P^M_{II}}{P^M_{III}} = \frac{B^N(i_{III} - 1) + 1}{B^M - 1} = \frac{1{,}614(1{,}632 - 1) + 1}{0{,}252 - 1} = 2{,}7 < 0$$

$$\frac{P^N_{II}}{P^N_{III}} = \frac{i_{III}}{1 - \frac{1}{B^N}} - 1 = \frac{1{,}632}{1 - \frac{1}{1{,}614}} - 1 = 3{,}4 > 0$$

Therefore the direction of the performance-flow is shown by the Fig. being in the third column of the second row of Table 3 and the arrows were drawn into Figs. 58 and 59 on the base of this.

From the examples can be seen that in the mechanical change-speed gear with epicyclic gear for connecting element generally a clutch (brake) is used (for simply fastening the epicyclic gear) on the one hand, while on the other a further epicyclic gear or epicyclic gears (for the transfastening), previously, of course, transformed to one with one degree of freedom.

In connection with the latter it is also obvious that if all of the connecting elements used for the transfastening are epicyclic gears (of whatever type), then the role of the epicyclic gear to be fastened and of the one used as connecting element can be changed with each other. In Fig. 60 e.g. such an epicyclic gear prefastened by epicyclic gear N can be seen at which shaft III of epicyclic gear N is back-fastened to the shaft of M by the simply fastened epicyclic gear O.

This, otherwise, is in harmony also with that what was said in connection with Fig. 56 about the complex epicyclic gear with changed shaft.

Fig. 60, therefore, can be constructed also according to Fig. 61.

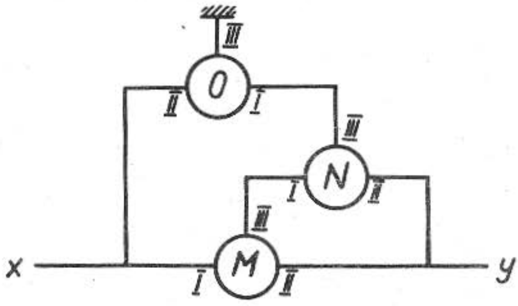
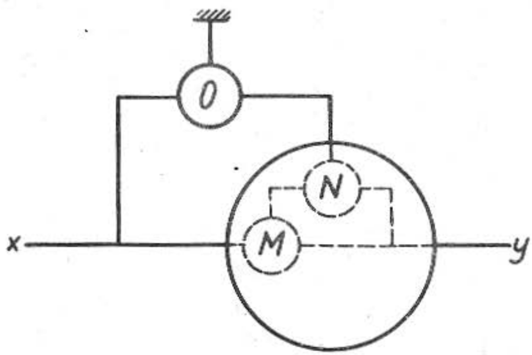


Fig. 60.

Summing up, it can be stated that in all change-speed stages of a given transmission epicyclic gears of optional number can take part in the power transmission, however, at least one epicyclic gear-member must be connected to the incoming shaft and one epicyclic gear-member to the base. As a matter of course, these epicyclic gear members do not have to pertain unconditionally to the same epicyclic gear. When increasing the number of the epicyclic gears, therefore, the one member of the newer epicyclic gear must be connected to the epicyclic gear member previously intended for fastening, an other member of it to some other member of the formerly existing epicyclic gear (or gears) and its third member will be simply fastened, resp. if we want to multiply with further epicyclic gear the change-speed gear stage, then we connect to this one member of the new epicyclic gear. With this new epicyclic gear in the followings we proceed in the same manner as was done with the previous one; we can connect newer and newer epicyclic gears up till fastening its last remaining member. After all, a change-speed gear stage of a given transmission can be realized with wpicyclic gear combinations of infinitive number. All combinations, however, can be led back to the three basic types described in the foregoing, so that in case the kinematical and dynamical correlations of the transfastened and crossfastened epicyclic gears are known, then all change-speed gear stages can be examined simply and in a manner easy to survey.



2. OUTSIDE ELEMENTS OF THE MECHANICAL CHANGE-SPEED GEARS WITH EPICYCLIC GEAR

The change-speed gears can be produced out of the diverse combinations of the change-speed gear stages dealt with in the previous paragraph.

Whatever kind of combination is selected, to the change-speed gear, in addition to the incoming and outgoing shafts, also brakes pertain, since at the change-speed gear purely with epicyclic gear at every stage an epicyclic gear has to be simply fastened in order to realize the transmission (to bind down the superfluous degree of freedom). The incoming and outgoing shafts as well as the brakes are called the outside elements of the change-speed gear (Fig. 62). Since the outside elements of the change-speed gear are in connection with each other only through the epicyclic gears, it is also only a two degree of freedom mechanism, independently from the number of its outside elements. Similarly, therefore, to the basic transmission of the epicyclic gear, the transmission of the change-speed gear also can be written. If we refer the angular speeds to the angular speed of the brake-drums, then the basic transmission is:

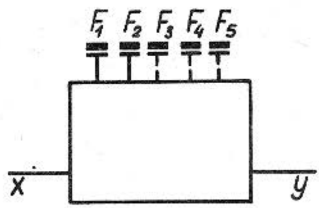


Fig. 62.

$$i_n = -\frac{\omega_y - \omega_n}{\omega_x - \omega_n} \tag{147}$$

where

ω_x — the angular speed of the incoming shaft,

ω_y — the outgoing shaft

ω_n — the angular speed of the brake drum pertaining to the n^{th} stage, where $n = F_1, F_2, F_3, F_4 \ldots\ldots$

If one of the brake drums is braked $(\omega_n = 0)$ we obtain the effective transmission:

$$i_n = \frac{\omega_y}{\omega_x} \tag{148}$$

Within the change speed gear the connection of the built in epicyclic gears (epicyclic gear members) with the outside elements of the change-speed gear can be constant or variable, more exactly permutable (rearrangable) and, accordingly, the change-speed gears with epicyclic gear can be distributed in two large groups (98):

a/ those with constant inside design,

b/ change speed gears with varying inside design.

3. CHANGE-SPEED GEARS WITH CONSTANT INSIDE DESIGN

31. Needed number of epicyclic gears

All three members of any epicyclic gear are in constant connection with the outside elements of the change-speed gear, according to some combination. The only manner of change-speed: of the brake drums one must be stopped, that is, the superfluous degree of freedom of one of the epicyclic gears has to be bound down. This means, that to an epicyclic gear of n stage pertain n pieces of brake drums, and to each brake drum an epicyclic gear, or in other words:

into the change-speed gear with epicyclic gear and of constant inside design as many epicyclic gears have to be built in as many stages differing from 1:1 are intended to be realized.

After all these, let us examine the possibilities for producing change-speed gears of the n stage.

32. One stage change speed gears

Of the change-speed gear stages with epicyclic gear dealt with in paragraph 1. theoretically any one can be selected, for every one of them presents a single constant transmission. According to the abovesaid, however, for stage $n = 1$ is a single epicyclic gear sufficient. In Fig. 63 the scheme of the one stage change-speed gear can be seen.

It is a matter of course that also the "direct" transmission (1:1) can be realized in any change-speed gear with epicyclic gear, for this only the connecting (blocking) of two of the change-speed gear's outside elements is needed. The three outside elements of the one stage change-speed gear present three possibilities of blocking. In Fig. 64 all three possible locations of the short-closing clutch was drawn with dashed line.

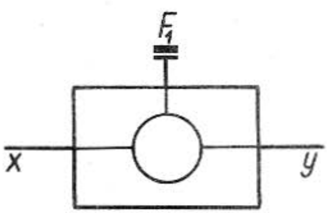


Fig. 63.

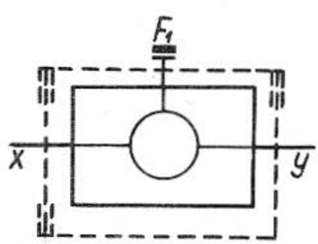


Fig. 64.

33. Change-speed gears of two stages

The needed and necessary number of epicyclic gears is two. For designing the change-speed gear with two epicyclic gears, neverthless, several combinations are possible.

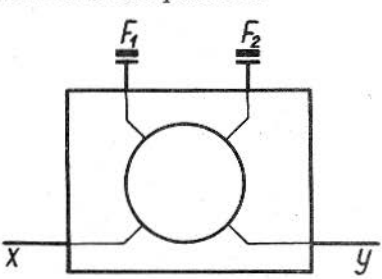
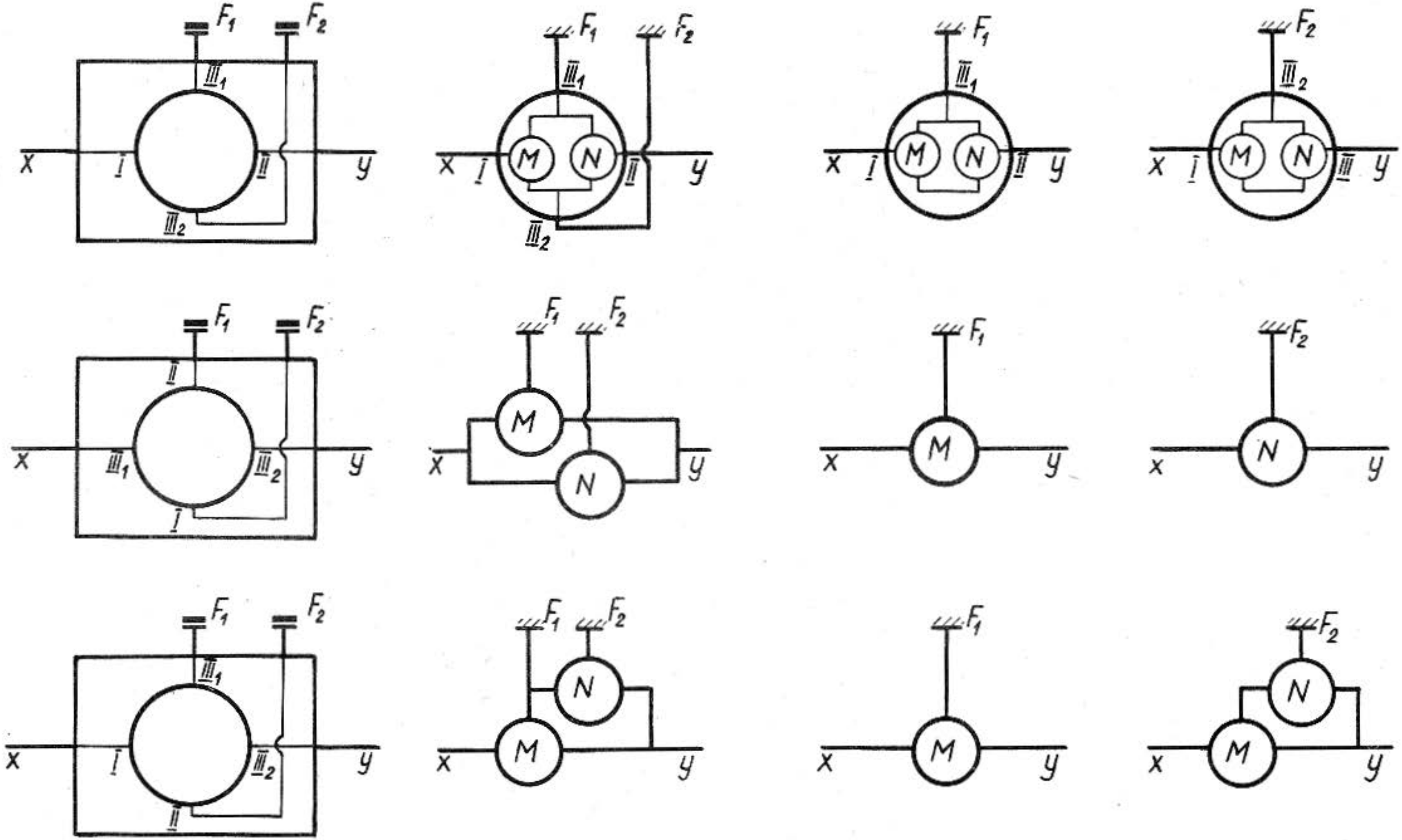


Fig. 65.

To deduct the individual combinations, we start from the complex epicyclic gear. As already described in the abovesaid, the complex epicyclic gear has four shafts, the I and II, as well as the two III (III_i and III_2).

The change-speed gear with two stages has also four outside elements, to these have each shafts of the complex epicyclic gear to be connected. (Fig. 65.). For fastening-in the complex epicyclic gear and together with


F1
F2
X
y
I
II
III1
III2
III
M
N

this for the designing of the change-speed gear with two stages, three variations are possible:

The complex epicyclic gear will be fastened-in in straight position, in inverted position or in straight position, but with changed shafts II - III_2. The latter possibility was already spoken of in point 1 of the present paragraph, when the interpretation of the epicyclic gear transfastened by the simply fastened epicyclic gear was set forth.

All three variations are presented in Fig. 66, where the individual stages can be seen also separately.

The first variation's both stages are simply fastened epicyclic gears and the two stages are obtained by fastening one or the other of shafts III.

The second variation consists, as a matter of fact, of two parallel epicyclic gears, and the two stages are obtained so that once the first, and then the other epicyclic gear is fastened, while the epicyclic gear not fastened runs freely.

One stage of the third variation is the simply fastened epicyclic gear, while the other is the transfastened one.

All three variations were deducted for straight drive. In case of reversed drive no change at all will occur at variations first and second and also at the third variation only as much, that the prefastened epicyclic gear will be transformed to a back-fastened one.

34. Change-speed gears of three stages

The needed and sufficient number of epicyclic gears is three. For designing a change-speed gear with three epicyclic gears several combinations are available.

For the deduction of the individual combinations we are starting from the doubly complex epicyclic gear. The doubly complex epicyclic gear consists of a complex- and a simple epicyclic gear, so that the complex epicyclic gear will be transfastened or crossfastened by the simple epicyclic gear, though, all shafts will be led out also further.

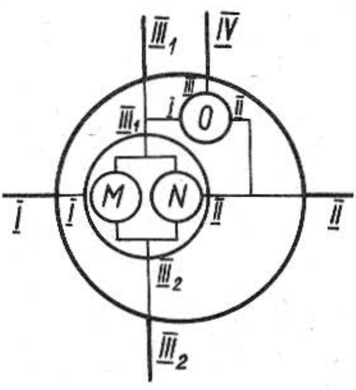


Fig. 67.

The same process will be performed also then, when the complex epicyclic gear is transfastened to a one degree of freedom one, but then the shafts which became kinematically too determined, are not led out, since this is not needed (Figs. 34, 35 and 36).

At present all shafts are led out, since just by the variations of the fastenings of these do we obtain the individual stages.

For the doubly complex epicyclic gear also three Figs. are obtained because at crossfastening, the complex epicyclic gear can be straight and inverted as well.

The characteristics of the three variations are:

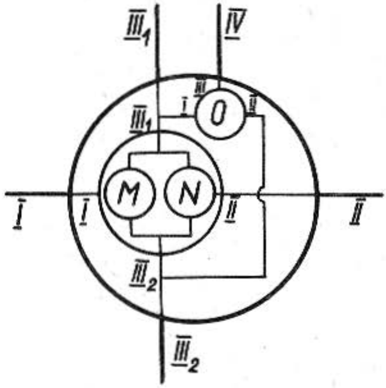


Fig. 68.

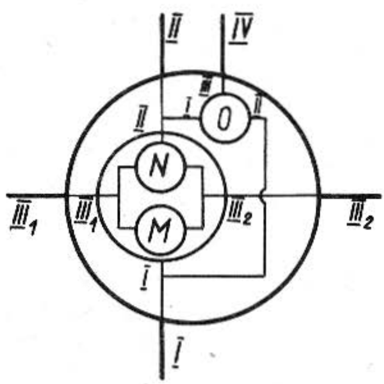


Fig. 69.

A. Of the five shafts two are in direct connection with an epicyclic gear each, two shafts with two epicyclic gears each and one with three epicyclic gears (Fig. 67).

B. Of the five shafts three are in direct connections only with one simple epicyclic gear each and the other two shafts with every one of the three simple epicyclic gears (Fig. 68.).

C. Of the five shafts one is in direct connection with one epicyclic gear, while four shafts with two epicyclic gears each (Fig. 69).

The third member of the simple epicyclic gear connected-on increases the number of shafts by one, so that the doubly complex epicyclic gear has five shafts; the I and II, further two III (III_1 and III_2) as well as the new shaft, marked III. The change-speed gear of three stages has also five outside elements (the incoming and outgoing shafts and three brakes), to these must a shaft each of the doubly complex epicyclic gear be connected (Fig. 70).

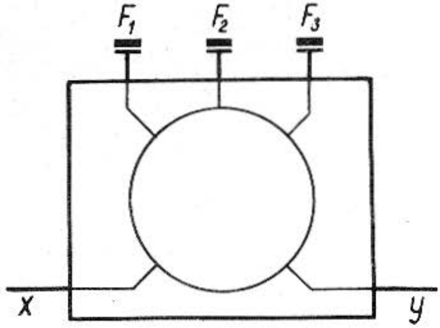


Fig. 70.

For connecting the shafts several variations are possible and each of these give a combination for the design of the change-speed gear of three stages.

Since the variation between each other of the connections with the brakes do not give a new combination, for fastening in their shafts, at all three doubly complex epicyclic gear types, theoretically only 20 variations are possible, resp. of the five shafts it is possible to select two shafts each for the purpose of connecting them to the incoming and outgoing shafts of the change--speed gear. For the change-speed gear of three stages, therefore, it would be possible to have 60 combinations in all. In fact, however, those shafts of the doubly complex epicyclic gear which are in direct connection with simple epicyclic gears of the same number, can be changed with each other, resp. are able to substitute each other without changing the theoretical design of the change-speed gear. Such are e.g. at type A the III_1 and I, also the II and III, at type B the III_1 and I, also the II and III, at type B the III_1 and III_3 etc.

It must be, further, taken in consideration that those variations at which the difference in only that, that the incoming and outgoing shafts are changed, i.e. they are each other's reciprocals, also can not be looked at as new types, as by changing the direction of drive one can be obtained from the other.

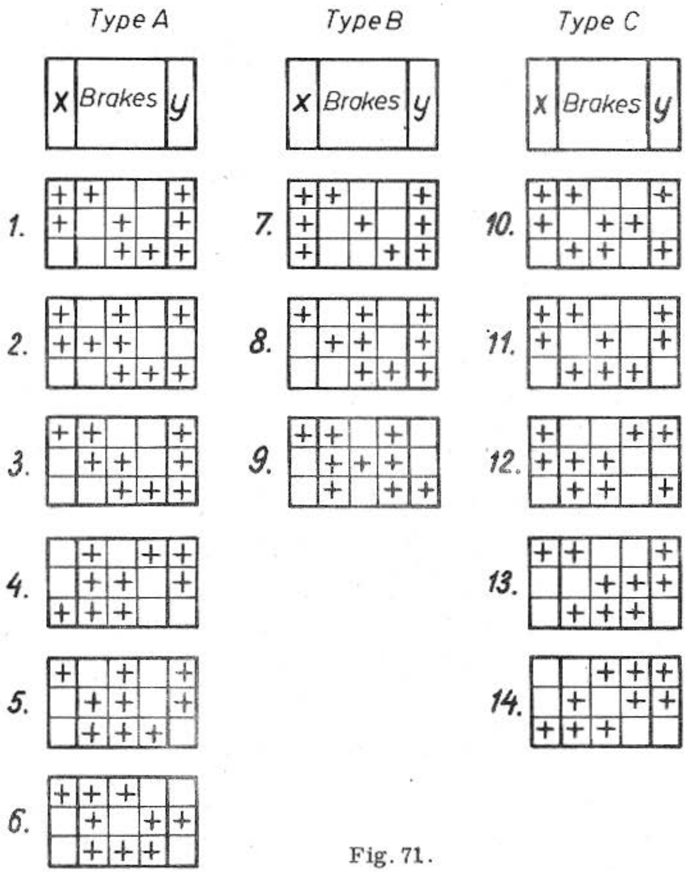
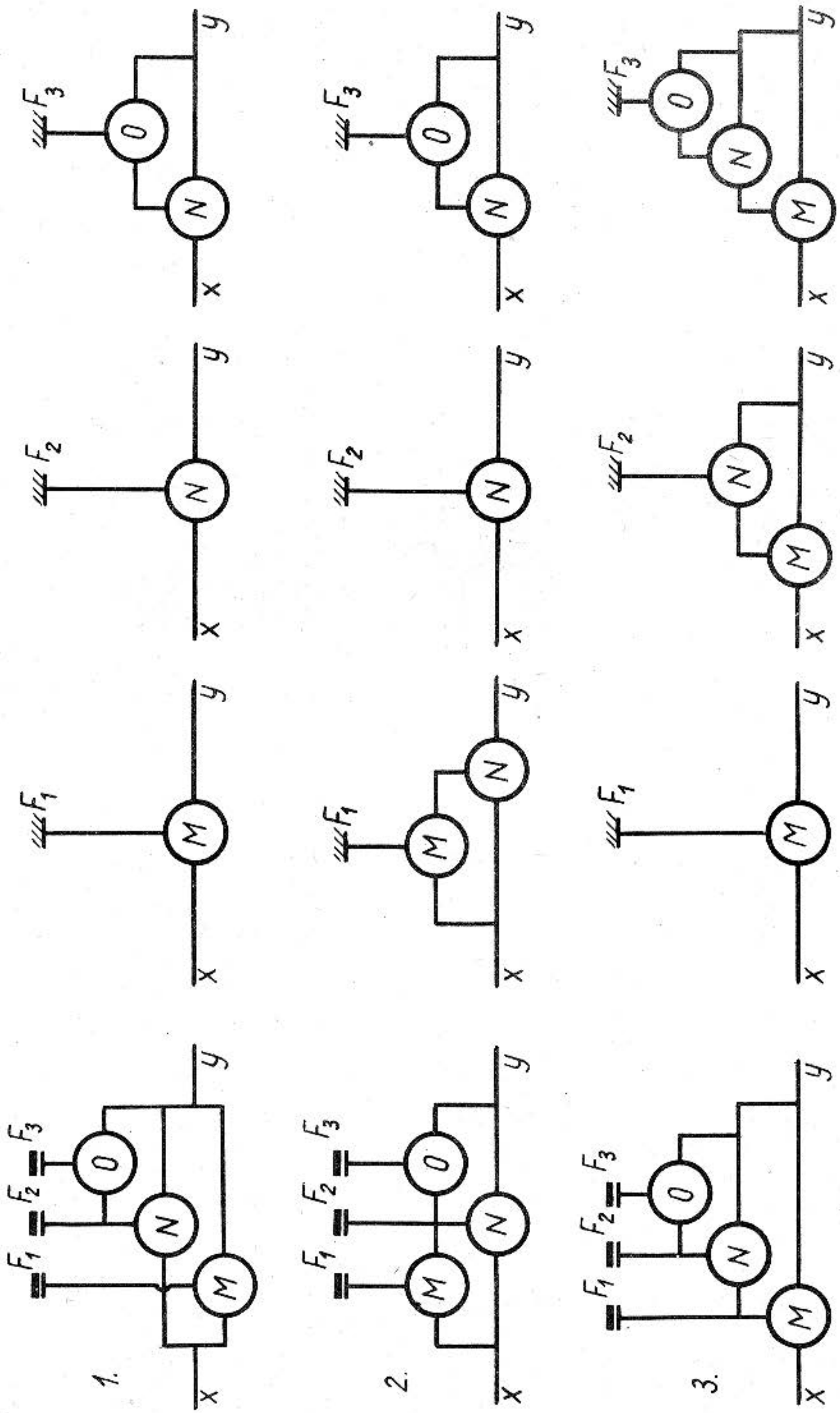


Fig. 71.

Practically, therefore, at type A doubly complex epicyclic gear instead of 20 in all 6 combinations, at type B 3, at type C 5 are possible, as it can also be seen on Fig. 71.

The Fig. well illustrates the connections of the individual epicyclic gear members with the shafts of the change speed gear. The + signs mean the direct connections. The horizontal lines correspond to an epicyclic gear each. Following from the foregoing the lines can be changed optionally. Of the vertical columns the two extreme ones correspond to the incoming and outgoing shafts of the change-speed gear, while the middle one to the three brakes.


1.
2.
3.
F_1
F_2
F_3
M
N
O
x
y

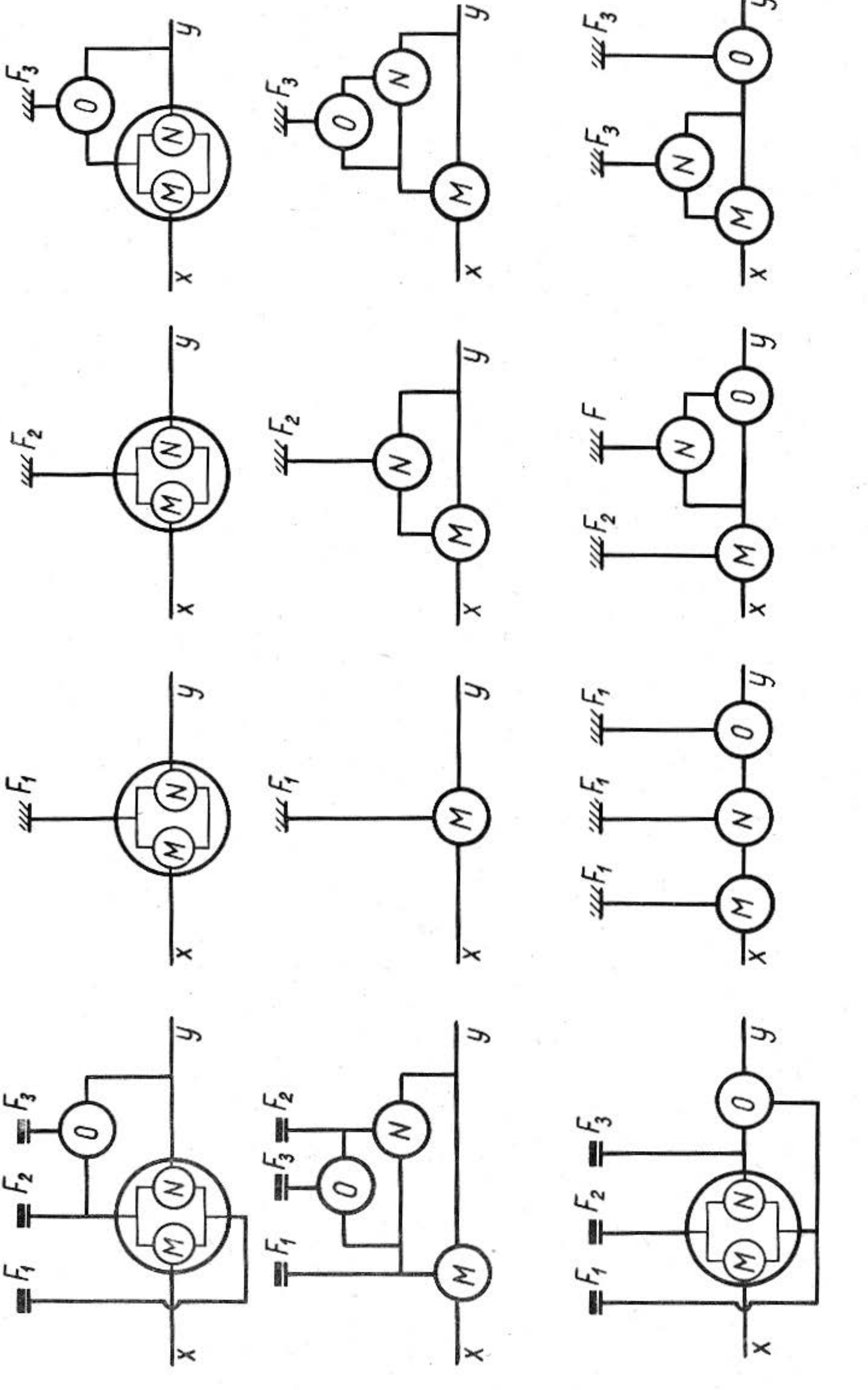


Fig. 72. b.

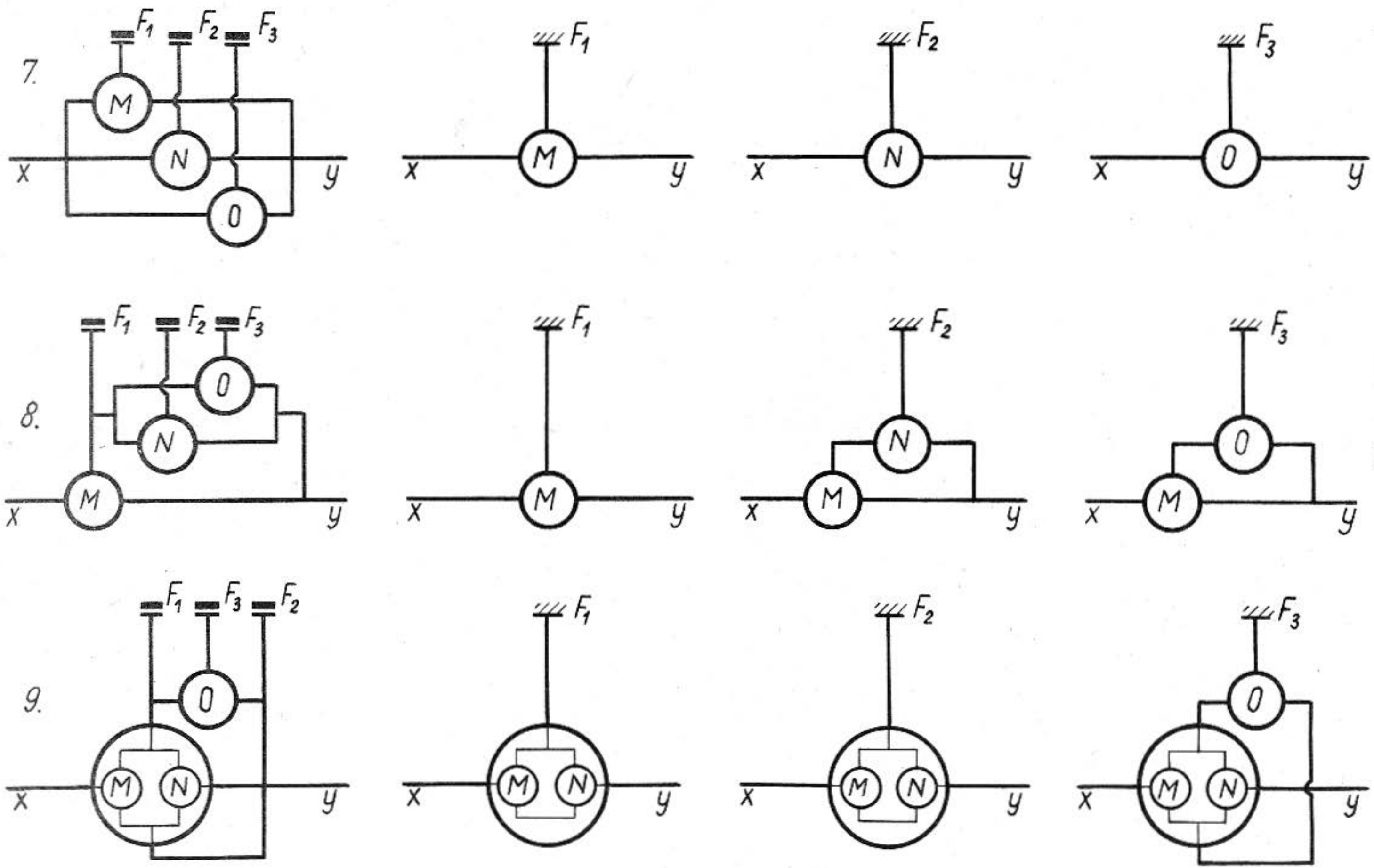


Fig. 72. c.

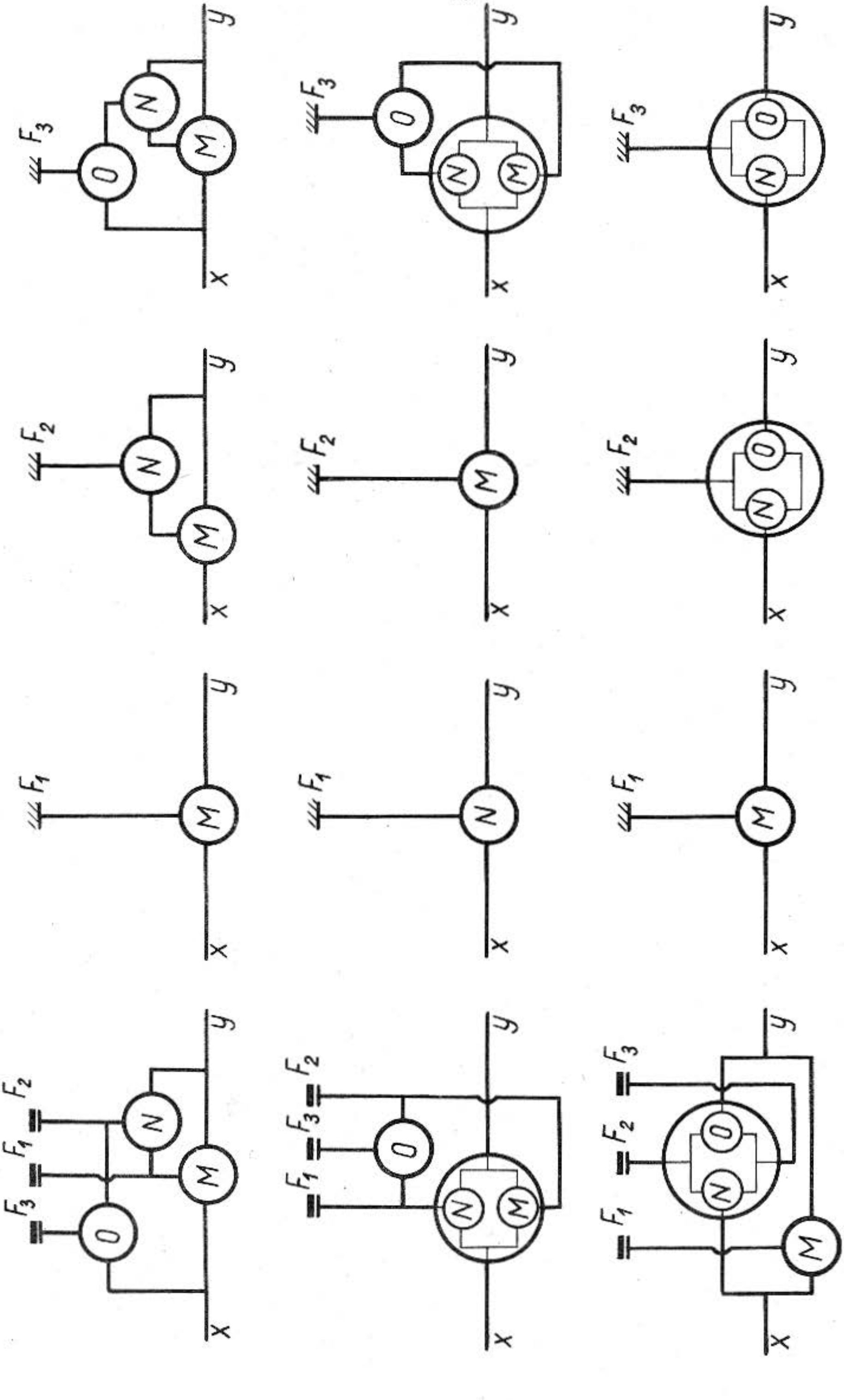


Fig. 72. d.

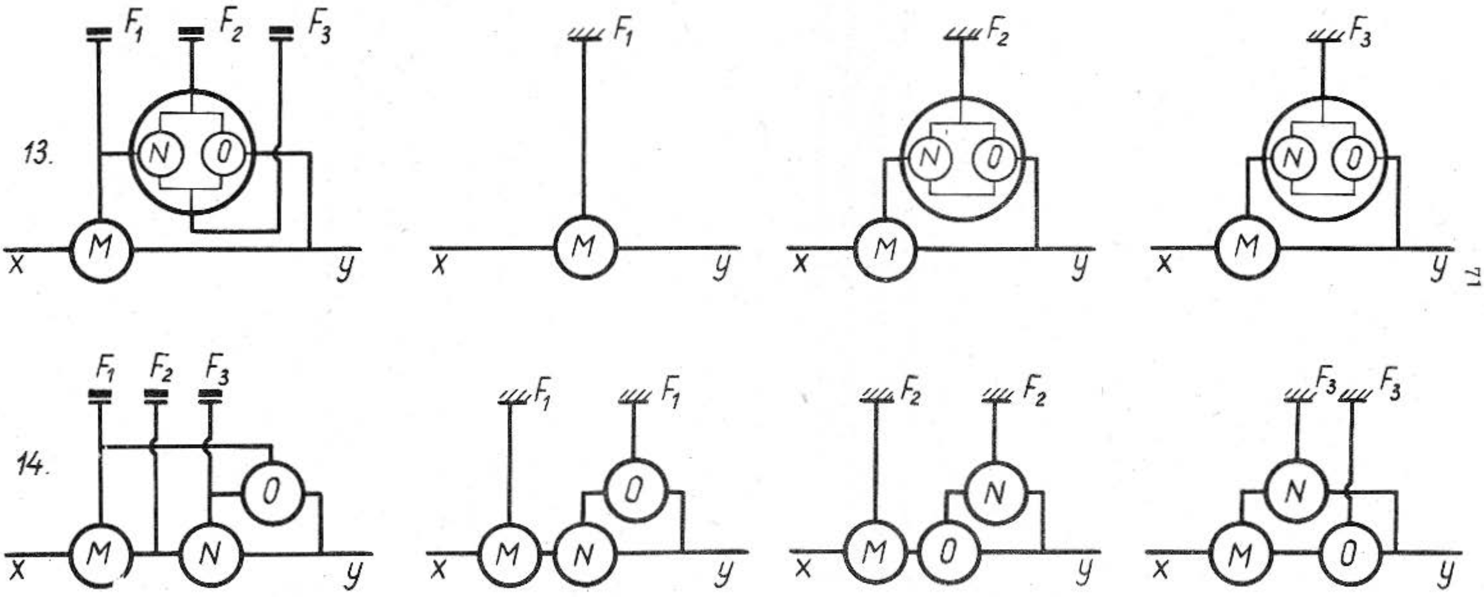


Fig. 72. e.

The three middle columns can be also optionally changed between each other, therefore letters F_1, F_2, F_3 were not indicated.

Outside of the 14 basic types shown in Fig. 71 no other change-speed gear of three stages with epicyclic gear and of constant inside design does exist. Therefore all change-speed gears, those to be found in the literature or to be plotted only in the future, can bel led back to one of the 14 basic types. Just for this, it is expedient to describe in details the 14 basic types (Fig. 72).

Of the individual change speed gear basic types, resp. of the stages of them the following can be stated, assuming straight drive:

Type 1 The first two stages are two simply fastened epicyclic gears, the third stage is an epicyclic gear prefastened by a simply fastened epicyclic gear.

Type 2 The three stages consist of an epicyclic gear backfastened by a simply fastened one, a simply fastened and an epicyclic gear prefastened by a simply fastened one.

Type 3 The first stage is a simply fastened epicyclic gear, the second an epicyclic gear prefastened by a simply fastened one, the third is a prefastened epicyclic gear, where the connecting element is an epicyclic gear transfastened by a simply fastened one.

Type 4 There is a complex epicyclic gear simply fastened in the first and second stage, in the third one prefastened by a simply fastened epicyclic gear.

Type 5 The first stage is a simply fastened epicyclic gear, the second and third a prefastened one, at the third stage, however, the connecting element is a transfastened epicyclic gear.

Type 6 In all three stages epicyclic gears are series connected: in the first stage three simply fastened, in the second and third one simply fastened and one epicyclic gear back- resp. prefastened by a simply fastened one.

Type 7 All three stages are simply fastened epicyclic gears, but at every stage another epicyclic gear will be simply fastened.

Type 8 The first stage is a simply fastened epicyclic gear, the second and third one prefastened by a simply fastened.

Type 9 The first and second stage are simply fastened epicyclic gears; once the one and then the other will be fastened. The third stage is a complex epicyclic gear crossfastened by a simply fastened one.

Type 10 The first stage is a simple simply fastened epicyclic gear, the second one prefastened by a simply fastened epicyclic gear, in the third stage an epicyclic gear transfastened by an epicyclic gear is backfastened by a simply fastened one.

Type 11 The first and second stages are simply fastened epicyclic gears, the third stage is an iverted complex epicyclic gear crossfastened by a simply fastened one.

Type 12 All three stages are simply fastened epicyclic gears, in the second and third stage, however, a complex epicyclic gear is simply fastened at its III_1, resp. III_2 shaft.

Type 13 The first stage is a simply fastened epicyclic gear, the second and third are prefastened epicyclic gears, where the connecting element is a simply fastened complex epicyclic gear.

Type 14 In the first and second stages a simply fastened epicyclic gear and an epicyclic gear transfastened by a simply fastened epicyclic gear are series connected. In the third stage an epicyclic gear prefastened by a simply fastened one can be seen, where on shaft II also a connecting element can be found which is likewise a simply fastened epicyclic gear.

It must be noted that the sequence of the individual stages from viewpoint of the type has no significance whatever. At a concrete motor vehicle change--speed gear the sequence here taken at random is used to be grouped according to the increasing or decreasing row of the kinematical transmission.

35. Change-speed gears of four- and more stages

Already at the three stage change-speed gear a considerable number of combinations presented itself. In case of four- or more stages the number of combinations increases involution-like, there is no possibility any more to examine each individual combination, but this is needless at the same time, since of our examinations performed up till now the laws of design can also be ascertained and these can be presented in a table, resp. in a Fig.

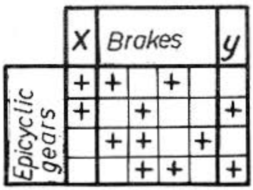


Fig. 73.

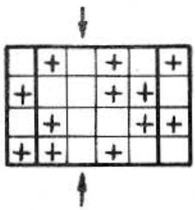


Fig. 74.

Fig. 73 shows a grid consisting of n lines and n + 2 columns. (In given case n = 4, but can also be more). The lines correspond to the epicyclic gears, while the columns to the outside elements of the change-speed gear. The sign + in the squares indicates the direct connection of the epicyclic gear to the given outside element of the change-speed gear.

In this grid can be drawn in all variational possibilities of the binding-in of the n pieces of epicyclic gears pertaining to the change-speed gear of n stage, according to the following rules:

1. In each line at least and max. three + signs will be drawn, since all epicyclic gears have three members.

2. The + signs will be distributed so that into each column should be placed at least one + sign.

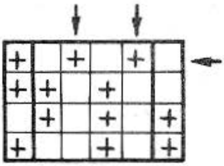


Fig. 75.

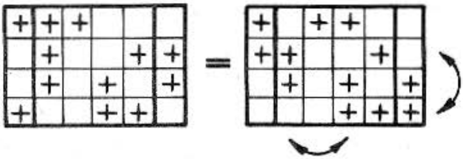


Fig. 76.

3. In two or more columns only than can a + sign stand in itself if they are not in the same line (Fig. 75 e.g. is not adequate!).

4. Now let us transplant all + signs individually in an other square, taking the foregoing three rules in consideration. If by the change of the changeable columns can not the original picture be reestablished, then we got a new combination. The lines, resp. the columns pertaining to the brakes, namely, can be freely changed between each other, this does not change the theoretical design (Fig. 76) of the change-speed gear i.e. such combinations do not lead to a new change-speed gear combination. (In Fig. 77 e.g. the original picture can be reestablished, in Fig. 75 not any more.)

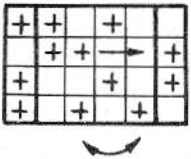


Fig. 77.

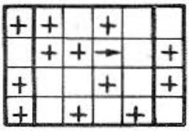


Fig. 78.

By means of the grid shown in Fig. 73 and of the taking in consideration the above four rules of variation, all combinations of any stage change-speed gears can be determined and examined.

In Fig. 79 the general theoretical scheme of the change-speed gear of four stages is presented. It follows from the foregoing, that the epicyclic gears indicated can be numbered into four large groups according to the circumstance: with which shaft of the change-speed gear are they in direct connection:

a/ with its both shafts (4 pieces)
b/ only with the incoming shaft (6 pieces)
c/ only with the outgoing shaft (6 pieces)
d/ with no shafts at all (4 pieces).

Consequently, of 20 kinds of epicyclic gears in all could be that four selected which we intend to use for the change-speed gear.

Analogously to Fig. 79 the general schematic figure of the change-speed gear of 5 or 6 stages also could be drawn, on which already 39 or 60 kinds of epicyclic gears should be indicated, of which the needed 5 or 6 epicyclic gears could be selected. Also from this may be seen how rapidly the number of combinations increases with the number of the stages. For practical purposes, however, this has no significance any more, and therefore it will not be dealt with here.

Let us mention once more, that the number of the stages of any kind of change-speed gear can be completed with one - with the "direct" - stage without applying a separate epicyclic gear. The transmission 1:1, namely, can be

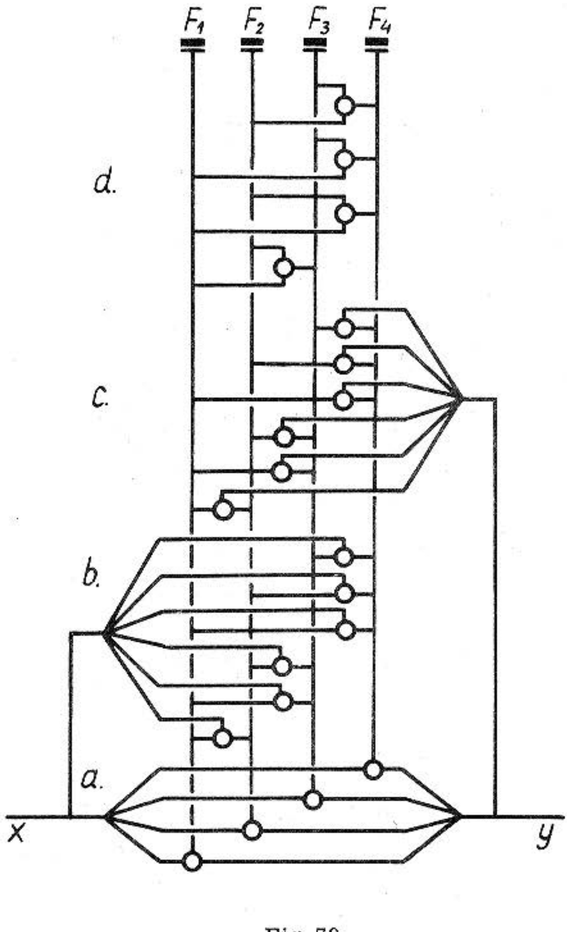


Fig. 79.

simply obtained, since only two of the outside elements of the change-speed gear need be closed short. Two viewpoints must be considered at selecting the main elements to be shortclosed:

a/ between which two outside elements (shaft, resp. brake drum) is the least torque difference,

b/ between two outside elements can the shortclosing element (e.g. frictional clutch) be built in most simply.

4. CHANGE-SPEED GEARS WITH VARYING INNER DESIGN

Let us return to the kinematical basic transmission of the simple epicyclic gear, which was marked by b. For binding-in the simple epicyclic gear - as it was dealt with in chapter II - there are 6 kinds of possibilities and according to these, for the general kinematical basic transmission (for the B) 6 kinds of values can be obtained at all individual simple epicyclic gears. Since at anyone of the change speed gear stages dealt with up till now by changing the value of B also the value of the transmission will be changed, it is obvious, that by varying the binding-in of one and the same simple epicyclic gear the number of the stages of the epicyclic gear can also be varied, resp. we can produce such an epicyclic gear at which the change-speed can be performed by varying the binding-in of the same epicyclic gear (permutation). Such a change-speed gear is called the change-speed gear with varying inside design. This means, that into the epicyclic gear with varying inside design less epicyclic gears are built than the number of such realizable stages the transmissions of which differ from 1:1. [100, 101] .

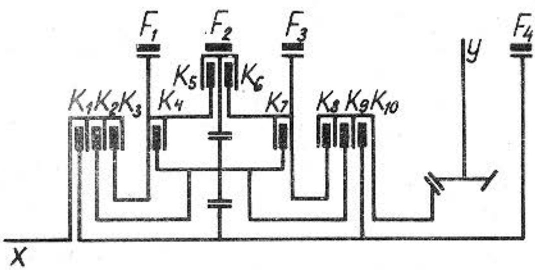


Fig. 80.

Theoretically from a one stage change-speed gear (simply fastened epicyclic gear) a change-speed gear of six stage can be developed, if the construction gives possibility for the individual fastening of all three members of the epiciclic gear, resp. for connecting them to the incoming and outgoing shafts. The change-speed gear of six stages built upon this principle would utilize up to 100 per cent the possibilities inherent in the epicyclic gear; this would be, however, a structure rather complitated.

At the execution shown in Fig. 80 e.g. four brakes, four simple-action and two treble-action, essentially, therefore, ten clutches would be needed. A further difficulty is due to the fact that from the outgoing shaft only by a gearing can the drive be transmitted. At the individual stages the brakes, resp. clutches enumerated in the following Table have to be coupled:

	To the incoming shaft	To simply fastening	To the outgoing shaft
1	K_1	$K_4 + F_1$	$K_6 + K_8$
2	K_1	$K_5 + F_1$	K_9
3	K_2	F_4	$K_6 + K_8$
4	K_2	F_2	K_{10}
5	$K_3 + K_5$	$K_7 + F_3$	K_{10}
6	$K_3 + K_5$	F_4	K_9

The epicyclic gear utilized up to 100 per cent, therefore, demands an exaggerated number of complementary structures (coupling elements), moreover, it is not probable that all the six transmissions meet the requirements raised by the aim of empolyment. There is not too much possibility for varying the transmissions, since the selection of a single transmission determines also the other five, though it may be anyone of the six. By decreasing the number of the variable possibilities of binding-in also the number of the coupling elements (clutches, brakes) decreases. The optimal proportion will be obtained, when for the coupling to the incoming and as well to the outgoing shafts the actuation of a single clutch is sufficient and the brake drums are in constant connection with the main elements to be disengaged, as it can also eb seen e.g. in Fig. 81. Here by the single epicyclic gear 3+1 transmissions are realizable by actuating the following coupling elements:

In stage 1:	$K_1 + F_1 + K_4$
In stage 2:	$K_2 + F_1 + K_5$
In stage 3:	$K_3 + F_2 + K_5$
In direct:	$K_1 + K_2 + K_4$

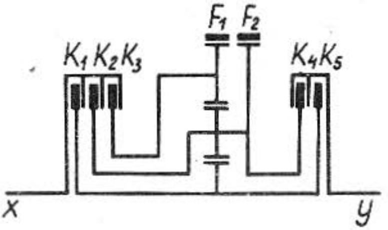


Fig. 81.

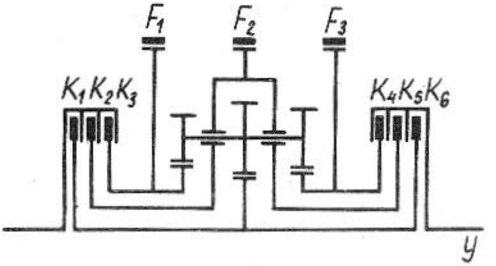


Fig. 82

The number of stages can be increased by using not a single simple, but more or a complex, perhaps conjunct epicyclic gears.

By the complex epicyclic gear shown in Fig. 82 e.g. nine transmissions can be realized. At the change-speed gear consisting of two epicyclic gears and shown in Fig. 83 already 18 kinds(!) of transmission can be obtained. At such possibilities it is rather probable that among the 18 we find 5-7 such stages which practically meet the requirements, the more so, as here the value of two of the 18 is freely chosen and namely of any two of the 18. Since by an epicyclic gear of any kind dealt with in chapter I, at any change--speed gear stages can the to a

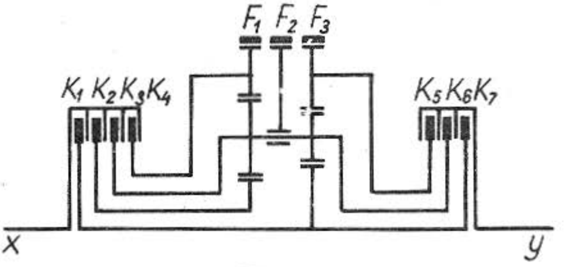


Fig. 83.

certain extent variable binding in be realized, the number of the producible combinations of epicyclic gears with varying inside design is exceedingly great.

Finally let us mention that at these change-speed gears for the realization of the direct stage no separate short-closing coupling element is needed, since the built in coupling elements already give possibility for connecting to the incoming or outgoing shafts not only a single epicyclic gear member each, but simultaneously two and this already means the short-closing of the change-speed gear.

5. CHANGE-SPEED GEARS OF TWO MEMBERS

Of the different combinations of the change-speed gears of constant and varying inside design a newer type of change-speed gear can be produced. These change-speed gears are developed not of combinations of change-speed gear stages but of that of change-speed gears, and, therefore, they are called multi-stage change-speed gears. Practically it is sufficient to examine the change-speed gears of two members. The change-speed gear of two members is produced by series-connecting two independent change-speed gears (Fig. 84). Since the transmissions of the two change-speed gears give together the transmissions of the change-speed gear of two members, the member of stages of the two members separately and also together is less, than that of the change-speed gear of two members. This means that still more stages can be realized by less epicyclic gears.

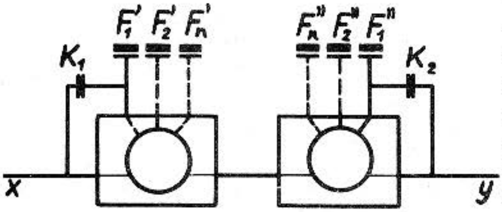


Fig. 84.

For the change-speed gear of two members the following combinations are possible:

1. both members are change-speed gears of constant inside design;
2. one member is a change-speed gear of constant inside design, while the other of a varying one;
3. both members are change speed gears of varying inside design.

At the change-speed gears of two members the direct stages have special significance, as they are giving possibility for that, that the modification of the members should be effective also separately.

Otherwise, it is charasteristic for the change-speed gears of two members that always the simultaneous connecting-in of elements is needed.

The number of the possible stages of the change-speed gear of two members, in adition of the direct, is:

$$n = (n_a + 1)(n_b + 1)$$

where n_a and n_b is the number of the stages of the members (in addition to the direct). In the practice the following cases can present themselves:

1. two members of one stage:

$$n_a = 1$$

$$n_b = 1$$

The member of the stages of the change-speed gear of two members is: 3, in addition to the direct (Cotal). The individual transmissions are:

$$i_I = i_a \quad i_b$$

$$i_{II} = i_a$$

$$i_{III} = i_b$$

$$i_D = 1$$

2. The one member is of one stage, the other of two:

$$n_a = 1 \qquad\qquad n_b = 2$$

The member of the stages of the change-speed gear of two members is 5, in addition to the direct (e.g. Fig. 85). The individual transmissions:

$$i_I = i_a \quad i_{b1}$$

$$i_{II} = i_a \quad i_{b2}$$

$$i_{III} = i_a$$

$$i_{IV} = i_{b1}$$

$$i_V = i_{b2}$$

$$i_D = 1$$

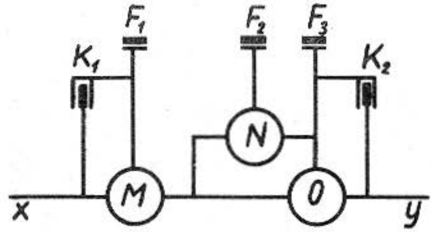


Fig. 85.

3. Two members of two stages:

$$n_a = 2$$

$$n_b = 2$$

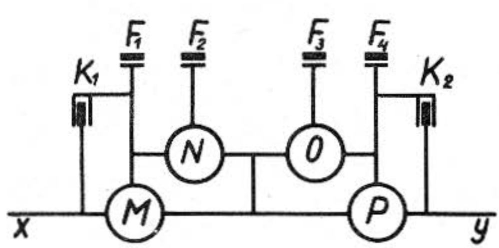


Fig. 86.

The member of the stages of the change-speed gear of two members, in addition to the direct, is: 8 (e.g. Fig. 86). The individual transmissions:

$$i_{I} = i_{a1}\ i_{b1} \qquad i_{IV} = i_{a2} \qquad i_{VII} = i_{b1}$$

$$i_{II} = i_{a1}\ i_{b2} \qquad i_{V} = i_{a1} \qquad i_{VIII} = i_{b2}$$

$$i_{III} = i_{a2}\ i_{b1} \qquad i_{VI} = i_{a2} \qquad i_{D} = 1$$

4. The one member is of two stages, the other has more stages than this:

$$n_a = 2 \qquad n_b = 3 \sim 6$$

At such multi-stage change-speed gears the two members are eventually already structurally separated (the member of two stages is called then cross--country change speed gear too).

Of the change-speed gear of two members can also be told, just like of the change-speed gears with varying inside design that all of their transmissions are not always utilizable. E.g. at motor vehicles the individual transmissions must be in a certain connection with each other and in case of anyone of the transmissions not fitting into this row, this will be left out of the coupling. In spite of this the change-speed gears of two members are rather well utilizable since by applying them epicyclic gears can be saved.

IV. HYDROMECHANICAL CHANGE SPEED GEARS WITH EPICYCLIC GEAR

1. THE COMPLEX EPICYCLIC GEARS AS HYDROMECHANICAL CHANGE-SPEED GEAR STAGES

11. The simply fastened epicyclic gear as hydromechanical stage

The complex epicyclic gear is called then a hydromechanical drive stage when it has at least one connecting element which is some kind of hydraulic power-transmission structure (hydrodinamical clutch or torque converter, eventually a hydrostatic drive).

Of the three connecting elements of the simply fastened epicyclic gear theoretically any one resp. any number of them can be a hydraulic clutch or torque converter. In the practice, however, only that case is interesting when either into the O or the ∞ shaft is the connecting element inserted, while the i_{III} is a simple mechanical clutch (brake) as it can be seen in Fig. 87. resp. in Fig. 88.

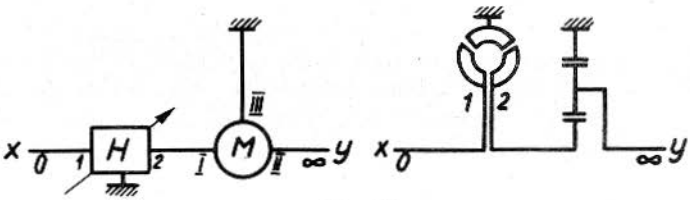


Fig. 87.

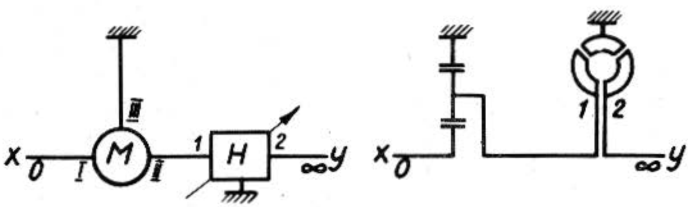


Fig. 88.

The complex epicyclic gear fastened in this manner is, after all, the series connection of a hydraulic element and a mechanical transmission. Such hydromechanical stages are rather widely used in the change-speed gears of motor vehicles.

The characteristics of the stages can be simply determined by means of the previously given formulae.

The kinematical transmission is in general:

In our case the value of i_∞ (Fig. 87) resp. of the i_o (Fig. 58) is equal to 1, while the transmission of the other connecting element is equal to the hydraulic transmission, therefore, for both cases can be written that

$$i = i_H B \tag{149}$$

The formula of the torque transmission is:

$$k = -\frac{k_H}{B} \tag{150}$$

It has no reason to examine the efficiency at the simply fastened epicyclic gear considered that the losses of the epicyclic gear itself will be neglected (ideal epicyclic gear), therefore:

$$\eta = \eta_H$$

In case of automatic connecting elements - such are the hydrodinamical torque converter or the clutch - also the extent of the taken torque must be examined, since this varies dependent on the kinematical conditions. The torque taken by the hydrodinamic unit itself is characterized by the factor λ_{sz} [33, 35, 37, 39 etc.] while the torque taken by the complex epicyclic gear by λ_x . This factor λ_x in the above two cases will not be uniform.

According to Fig. 87 the drive acts directly on the shaft, the torque taken is therefore:

$$M_x = M_{sz} = \gamma \lambda_{sz} D^5 n_{sz}^2 \tag{151}$$

where

Y - the weigth of the liquid

λ_{sz} - is the factor of torque-take up of the hydraulic element, which is the function of $\lambda_{sz} = f(i_H)$

D - is the characteristic geometrical dimension of the hydraulic element (generally its diameter),

n_{sz} the rpm of the pump impeller.

Since the λ_{sz} can be formed similarly to λ_{sz}, we obtain

$$\lambda_{sz} = \frac{M_x}{\gamma D^5 n_x^2} = \lambda_{sz} \tag{152}$$

since

$$n_x = n_{sz}$$

On the other hand, in Fig. 88 the drive of the pump impeller is performed by modification, therefore the torque taken by the stage differs from the torque of the pump impeller, i.e.

$$M_x = B M_{sz} = B \gamma \lambda_{sz} D^5 n_{sz}^2 \tag{153}$$

Since

$$n_{sz} = B\, n_x \tag{154}$$

therefore

$$M_x = +B^3 \gamma\, \lambda_{sz}\, D^5\, n_x^2 \tag{155}$$

The factor of proportion, therefore is

$$\lambda_x = \frac{M_x}{\gamma\, D^5\, n_x^2} = B^3\, \lambda_{sz} \tag{156}$$

For drawing the outside characteristics of the hydromechanical stage - as can be seen - the $\lambda_{sz} = f(i_H)$ relation should be known, more correctly speaking the outside characteristics of the hydraulic element. This is understandable seeig that the hydromechanical element fundamentally retains the properties of the hydraulic element, the mechanical part - the epicyclic gear - can only form, modificate this [39].

For the sake of illustration let us take a sample-characteristic for the hydraulic element and calculate with this. This method gives possibility for drawing further conclusions.

At the hydrodinamical clutch the outside characteristic consists in its essence only of $\lambda_{sz} = f(i_H)$ For sake of simplicity let be

$$\lambda_{sz} = \lambda_o (1 - i_H^4) \tag{157}$$

The characteristic can be seen in Fig. 89. where also the efficiency is indicated.

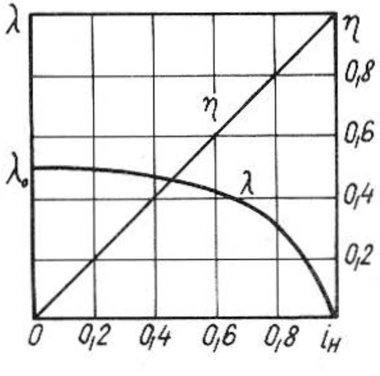


Fig. 89.

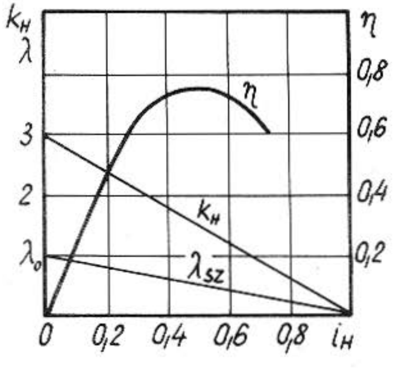


Fig. 90.

The outside characteristic of the torque converter contains more factors; as a minimum factor λ_{sz} and λ_T characterizing the torque of the pump impeller and the turbine wheel. For practical reasons instead of λ_T the torque transmission $\lambda_T / \lambda_{sz} = k_H$ is used to be given. The efficiency curve also from these can be determined ($n = -k_H\, i_H$).

In Fig. 90 the formulae of the two basic characteristics can be seen:

$$\lambda_{sz} = \lambda_o - i_H \tag{158}$$

$$k_H = 3(1 - i_H) \tag{159}$$

These sample-characteristics according to their character correspond to the characteristics of the clutches and torque converters used in the practice.

The characteristics seen in Fig. 91-94 were obtained on base of the sample-characteristics.

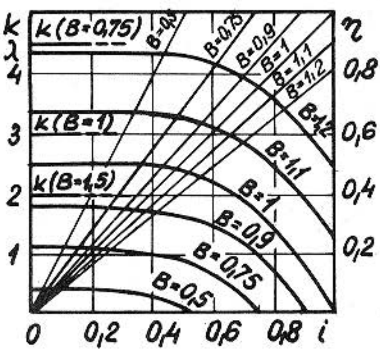


Fig. 91.

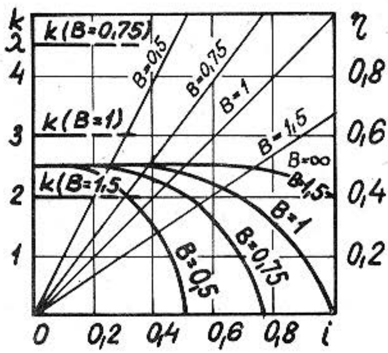


Fig. 92.

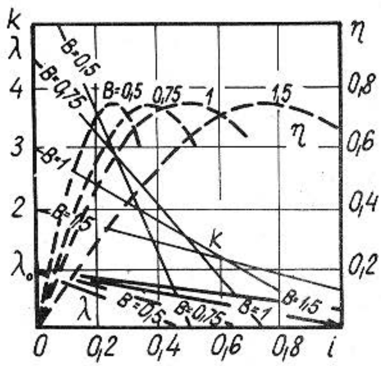


Fig. 93.

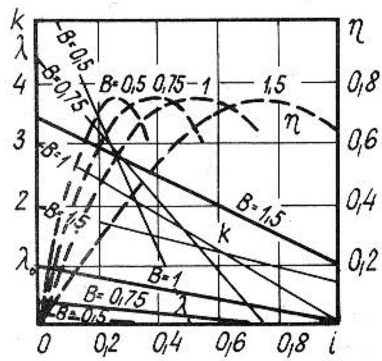


Fig. 94.

Figs 91 and 93 correspond to Fig. 87, therefore the hydraulic element is before the epicyclic gear. Of both characteristics can read off that the extent of the values λ and η do not depend on the value of B, only its place does. In other words, by varying B the transmission-range of the stage may be tightened or widened, while, of course, incraeses resp. decreases the torque transmission.

Figs. 92 and 94 correspond to the arrangement seen in Fig. 88, when the hydraulic element is after the epicyclic gear. This time the epicyclic gear influences already the incoming characteristics, i.e. beside tightening, resp. widening, the transmission range also decreases, resp. increases the extent of the incoming torque. This can be said in other words also so, that the mechanical transmission connected before the hydraulic element has also such effect, as when the dimension of the hydraulic element would also be changed.

When such an effect is only partially needed, then the mechanical transmission can be bulit in distributed in two parts partly before, partly after the hydraulic element.

It has to be noted, that at the above examination in Fig. 87 and 88 beside the straight drive the binding-in of the hydraulic element was also performed straightly. There is, of course, also possibility for the reversed binding-in of the hydraulic element (when the pump impeller is on the 2 side, the turbine wheel on the 1 side), at the simply fastened epicyclic gear, however, this is reasonable only then, when the drive is reversed too. In this case, however, we get back to the two cases dealt with up till this point.

12. The transfastened epicyclic gear as hydrodinamical stage

121. General relations. The general scheme of the transfastened epicyclic gear is shown in Fig. 32. Connecting elements can be applied in four places in all.

Only the insertion of the hydraulic connecting element into shafts O or ∞ does not give a qualitative difference related to the simply fastened epicyclic gear, it results, however, a new type of hydraulic stage in the place of i_{II} or i_{III}. In the following, for the purpose of unambigouosness, we will put the first hydraulic element always in the place of i_{III}. The possibility is not excluded, that we should insert the hydraulic element in the place of i_{III} series connected with a mechanical transmission, which can be understood also so that we apply as connecting element a hydromechanical stage consisting of a simply fastened epicyclic gear (Fig. 87 or 88). Obviously, we can apply also more than one hydraulic element, e.g. also in place of i_{II}.

122. Basic types. Let us commence our examinations with the most simple case, when beside the hydraulic element there is no other connecting element, i.e. $i_0 = i_\infty = i_{II} = 1$ as it can be seen in Fig. 95.

Taking in consideration, that the drive can occur from two directions, further, that also the hydraulic element can be bound-in according to two manners, our examinations ought to be commenced at once for four types. In Fig. 96 are these four types illustrated:

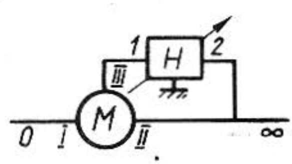
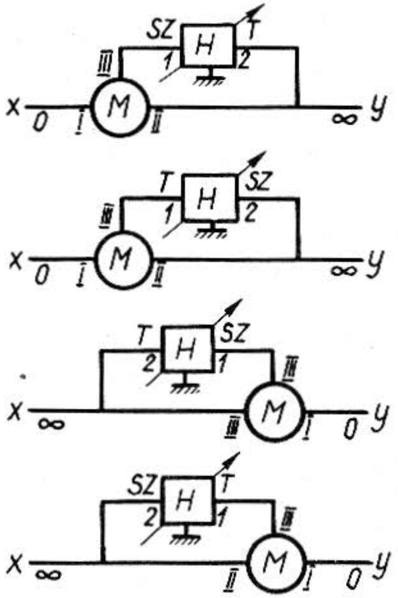


Fig. 95.



Type α : straight drive with a hydraulic element bound-in straight

Type β : straight drive, with a hydraulic element bound-in reversed

Type γ : reversed drive, with a hydraulic element bound-in straight

Type δ : reversed drive, with a hydraulic element bound-in reversed.

Fig. 96.

In the Fig. for all four cases a torque converter is indicated, since at this the change of the binding-in has greater significance. In case, namely, if the direction of the binding-in does not correspond to the direction of the performance-flow in that branch, then one can not calculate with a good efficiency from the outset, since the wheel designed as a pump impeller acts as a turbine, and conversely too. The direction of the performance-flow depends on the values of i_{II} i_{III} and B, more correctly on the value of the $\frac{i_{II}}{i_{III}}(1-B)$ expression.

Since in our case $i_{II} = 1$ the $i_{III} = i_H$ (in case of straight binding-in), resp. $i_{III} = \frac{1}{i_H}$ (in case of reversed binding-in) and from this follows, that in the operational range

$$0 < i_{III} < 1$$

or

$$i_{III} > 1$$

i.e. the value of i_{III} falls to a fixed interval, therefore the direction of the performance-flow depends mainly on the value of B. Practically, certain values of B, because of the direction of the performance-flow, demand a straight binding-in while its other values a reversed one.

This circumstance, otherwise, is clear in the following also from that, that in the got outside characteristics the values of B (as parameters) will figure only within certain limits in the operational range. The curves pertaining to the lacking B values are all positioned outside of the operational range, therefore they could not be indicated on the characteristics.

123. The kinematical characteristics. The examination of the epicyclic gear transfastened by the hydraulic element is started with the kinematical characteristics.

The general formula of the kinematical transmission takes the following form in our case for the four types:

type α :
$$i = \frac{1}{\frac{1}{B} + \left(1 - \frac{1}{B}\right) \frac{1}{i_H}} \tag{160}$$

type β :
$$i = \frac{1}{\frac{1}{B} + \left(1 - \frac{1}{B}\right) i_H} \tag{161}$$

type γ :
$$i = \frac{1}{B} + \left(1 - \frac{1}{B}\right) \frac{1}{i_H} \tag{162}$$

type δ :
$$i = \frac{1}{B} + \left(1 - \frac{1}{B}\right) i_H \tag{163}$$

For the sake of illustration it is worth while to show these relations on a diagram.

In Fig. 97 can be seen that at type α the curves pertaining to the B = = O 1 values are going outside of the operational range. Naturally, in case of reversed binding-in just these are going inside the operational range (type β - Fig. 98). Otherwise, the operational ranges of type α and of the hydraulic element itself are conform to each other, while the operational range of type β beside the increasing B value, will be shortened so, that the beginning of the range gets far from i = O, just until the B value, i.e. $i_o = B$. This means practically that the hydromechanical stage of type β can not be used for the starting of motor vehicles, since also the outgoing shaft has to rotate already, with a certain rpm, when this stage will be coupled.

Of type α it is to be also noted that in case of $B = \pm \infty$ $i = i_H$, which corresponds to the case when there is no epicyclic gear, i.e. the performance goes through exclusively the hydraulic element.

At type γ - similarly to type β - the limits of the two operational ranges also do not correspond (Fig. 99), but here, in case of the values of B decreasing, the transmission range of the hydraulic element will be shortened, until the value $i_{HO} = \frac{1}{B}$

Otherwise at this type, in the operational range, an epicyclic gear of 1>B>O basic transmission can be used. Decreasing of the operational range of the hydraulic element results in that, that the commencing section of its characteristics, will be left out of account.

Type δ has the most simple kinematical characteristics, where the relation between the two transmissions is linear (Fig. 100).

A further advantage of it is, that both operational ranges can be decreased, but so, that the section lying outside of the operational range does not take a position too far. This means, that in certain cases - when the characteristics of the hydraulic element is advantageous e.g. in the vicinity of i_H / O - the outside characteristics of the hydromechanical stage can be improved. Otherwise outside of 1>B>O limits all B values pertain to the operational range.

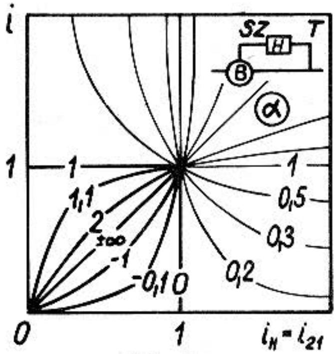


Fig. 97.

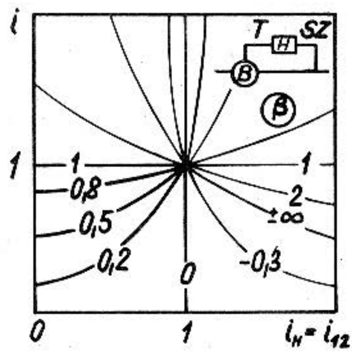


Fig. 98.

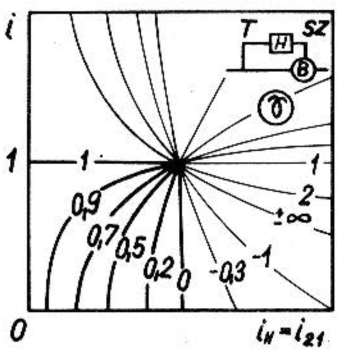


Fig. 99.

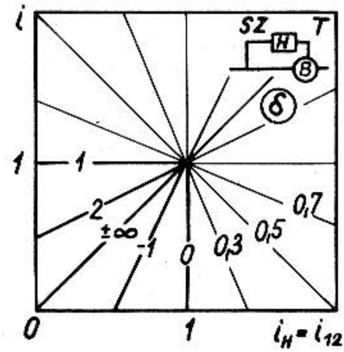


Fig. 100.

Also at type δ there is such a B value - similarly to type α - when the epicyclic gear's role will be superfluous ($B = \pm\infty$) and the hydraulic element remains alone.

Otherwise at all four types can be said that the hydraulic element will prevail the stronger, the nearer the kinematical characteristics proceed to the straight line (0,0) - (1,1).

124. <u>The outside characteristics.</u> After the examination of the kinematical characteristics let us determine the outside characteristics of the hydromechanical stage, i.e. the relations

$$a/\ k = f\ (i) \tag{164}$$

$$b/\ \eta = f\ (i) \tag{165}$$

$$c/\ \lambda_x = f\ (i) \tag{166}$$

which show in the function of the kinematical transmission the changes of the torque transmission, the efficiency and the torque take-up factor. The examination of the first two, as a matter of course, is reasonable only in case of a torque converter. Since, however, the outside characteristics of the hydraulic element are needed also here, we will use the sample-characteristics taken at the examination of the simply fastened epicyclic gears.

a/ The torque transmission. The torque transmission's (95) general formula takes the following form in our case for the four types: .

$$\text{type } \alpha \quad k = \left(1 - \frac{1}{B}\right) k_H - \frac{1}{B} \tag{167}$$

$$\text{type } \beta \quad k = \left(1 - \frac{1}{B}\right) \frac{1}{k_H} - \frac{1}{B} \tag{168}$$

$$\text{type} \quad k = \frac{1}{\left(1 - \frac{1}{B}\right) k_H - \frac{1}{B}} \tag{169}$$

$$\text{type } \delta \quad k = \frac{1}{\left(1 - \frac{1}{B}\right) \frac{1}{k_H} - 1} \tag{170}$$

We want, however, to obtain the torque transmission in the function of i. By using formulae (159) and (66) we obtain:

$$\text{type} \quad k = 3\left(1 - \frac{1}{B}\right)\left(1 - \frac{1 - \frac{1}{B}}{\frac{1}{i} - \frac{1}{B}}\right) - \frac{1}{B} \tag{171}$$

$$\text{type} \quad k = \frac{1}{\frac{3}{1 - \frac{1}{B}} - \frac{1}{\frac{1}{i} - \frac{1}{B}}} - \frac{1}{B} \tag{172}$$

$$\text{type} \quad k = \frac{1}{3\left(1 - \frac{1}{B}\right)\left(1 - \frac{1 - \frac{1}{B}}{\frac{1}{i} \; \frac{1}{B}}\right) - \frac{1}{B}} \tag{173}$$

$$\text{type} \quad k = \frac{1}{\frac{1}{\frac{3}{1 - \frac{1}{B}} - \frac{1}{\frac{1}{i} - \frac{1}{B}}} - \frac{1}{B}} \tag{174}$$

By means of the formulae one of the most important part of the characteristics, the curves of the torque transmission can be drawn (Fig. 101-104).

In Fig.101 the curves of type α may be seen. The original torque transmission of the torque converter pertains to $B = \pm\infty$. When the value of B is decreased then the characteristics originally straight become concave from below, the left-side end slides lower, while the right-side part of the curve approximates the ideal parabola. In case of increasing B value the opposite will

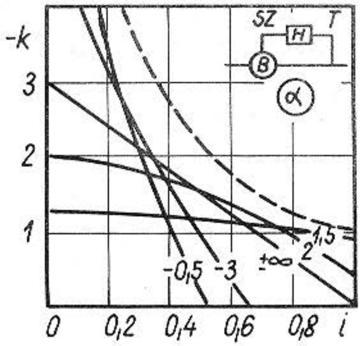


Fig. 101.

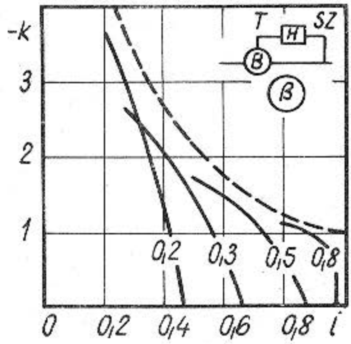


Fig. 102.

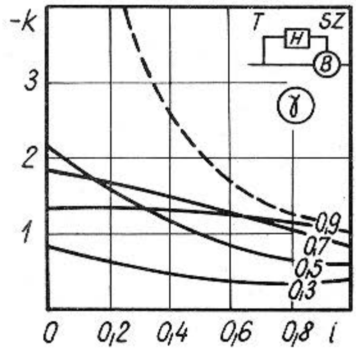


Fig. 103.

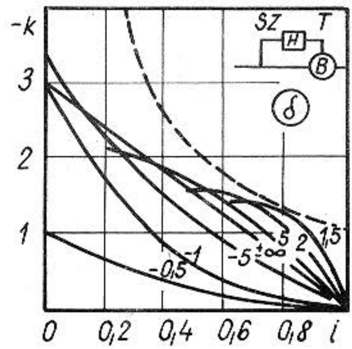


Fig. 104.

happen, the curve becomes convex and the right side part moves away from the hyperbola, moreover, when the negative B values are nearing the O, already the whole curve deviates to the left, resp. moves away from the hyperbola.

In Fig. 102 the decreasing of the operational range can be seen rather well. It can be seen as well, that at type β not beside any B value do we obtain the original characteristics of the torque converter, all curves are concave from below and generally exaggeratedly steep and also in their character they are far from the ideal hyperbola.

Fig. 103 differs basically from the former inasmuch that here the curves are generally exaggeratedly flat. It is their common property that in case of greater B value they are nearing very much the hyperbola in short section, which results in that place in good efficiency.

In Fig. 104 the characteristics of the type δ are shown, which is very similar to that of the type α. The basic difference lies in the shortening of the operational range. Moreover, such great torque modification can not be obtained by it as by type α, but the hyperbola is approached in a greater extent and in a longer section.

b/ The efficiency. From the characteristics of the torque modification can be derived the efficiency curves, since η = i k. Figs. 105-108 were plotted by utilizing Figs. 101 -104.

When comparing the four efficiency curves, the following evaluation can be given. Efficiency of the type γ is the least advantageous (Fig. 101) because it hardly rises over the straight line of 45^o which marks the efficiency of the clutch. It is obvious that it is not expedient in such case to apply the rather more complicated change-speed gear. The shape of the efficiency curve of the type α is more advantageous, but not as much as that of the type δ

The latter, namely, partly gives a better efficiency, partly this better efficiency is realized in a more wide interval. The shortening of the operational range which appears at the type δ does not represent any disadvantage in most cases. The shortening appearing at the type β, however, is of considerably greater extent, but it gives towards the middle of the operational range a rather good efficiency.

c/ The factor of torque take-up. An other decisive part of the outside characteristics is the curve of the factor of torque take-up, the $\lambda_x = f\ (i)$

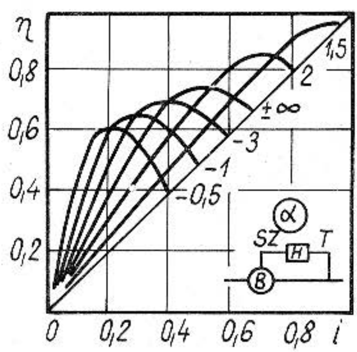


Fig. 105.

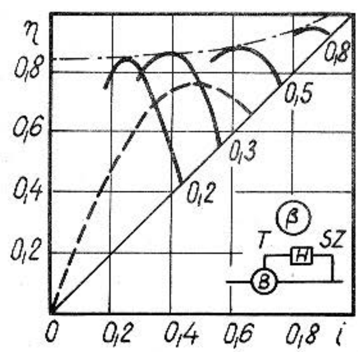


Fig. 106.

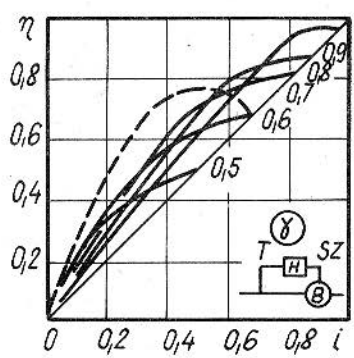


Fig. 107.

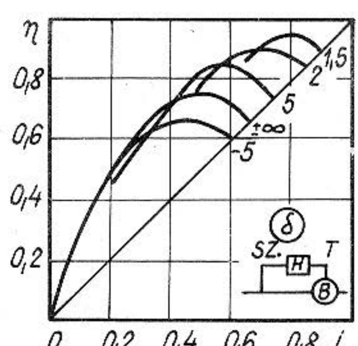


Fig. 108.

Considering that the torque take-up of the hydromechanical stage depends on the hydraulic element, at first the relation

$$\lambda_x = f\ (\lambda_{sz}) \tag{175}$$

must be determined. Let this relation be of the shape of

$$\lambda_x = \nu\, \lambda_{sz} \tag{176}$$

where ν is the proportion-factor which may depend on the value of B, i_H, k_H, i and k. For determining the ν let us write the torque appearing on the shaft.

At type α (Fig. 96), taking in consideration the expression (33):

$$M_x = M_0 = M_I = \frac{M_{III}}{\frac{1}{B}-1} = \frac{-M_{sz}}{\frac{1}{B}-1} = \frac{-\gamma\ \lambda_{sz} D^5 n_{sz}^2}{\frac{1}{B}-1} \tag{177}$$

Since

$$n_{sz} = \frac{n_y}{i_H} = n_x\ \frac{i}{i_H} \tag{178}$$

therefore

$$M_x = \frac{\gamma\ \lambda_{sz}\, D^5\, n_x^2}{1-\frac{1}{B}}\ \frac{i^2}{i_H^2} \tag{179}$$

From this

$$\lambda_x = \frac{M_x}{\gamma\ D^5\ n_x^2} = \frac{1}{1-\frac{1}{B}}\ \frac{i^2}{i_H^2}\ \lambda_{sz} = \nu\, \lambda_{sz} \tag{180}$$

wherefrom

$$\nu = \frac{1}{1-\frac{1}{B}}\ \frac{i^2}{i_H^2} \tag{181}$$

At type β :

$$M_x = M_I = \frac{M_{III}}{\frac{1}{B}-1} = \frac{-M_T}{\frac{1}{B}-1} = \frac{k_H\, M_{sz}}{1-\frac{1}{B}} = \frac{k_H\, \gamma\, \lambda_{sz}\, D^5\, n_{sz}^2}{1-\frac{1}{B}} \tag{182}$$

Since here

$$n_{sz} = n_y = i\, n_x \tag{184}$$

therefore

$$M_x = \frac{\gamma\, \lambda_{sz} D^5 n_x^2}{1-\frac{1}{B}}\ i^2 k_H \tag{185}$$

from this

$$\lambda_x = \frac{M_x}{\gamma D^5 n_x^2} = \frac{1}{1-\frac{1}{B}} i^2 k_h = \nu \lambda_{sz} \tag{186}$$

wherefrom

$$\nu = \frac{1}{1-\frac{1}{B}} i^2 k_H \tag{187}$$

At type γ :

$$M_x = \frac{M_y}{k} = \frac{M_{II}}{k} = \frac{M_{III}}{k\left(\frac{1}{B}-1\right)} = -\frac{M_{sz}}{k\left(\frac{1}{B}-1\right)} = \frac{\gamma \lambda_{sz} D^5 n_{sz}^2}{\left(1-\frac{1}{B}\right)k} \tag{188}$$

Since

$$n_{sz} = \frac{n_T}{i_H} = \frac{n_x}{i_H} \tag{189}$$

therefore

$$M_x = \frac{1}{1-\frac{1}{B}} \frac{\gamma \lambda_{sz} D^5 n_x^2}{k\ i_H^2} \tag{190}$$

and from this

$$\lambda_x = \frac{M_x}{\gamma D^5 n_x^2} = \frac{1}{1-\frac{1}{B}} \frac{1}{k\, i_H^2} \lambda_{sz} \tag{191}$$

wherefrom

$$\nu = \frac{1}{1-\frac{1}{B}} \frac{1}{k\ i_H^2} \tag{192}$$

At type δ :

$$M_x = \frac{M_y}{k} = \frac{M_{II}}{k} = \frac{M_{III}}{k\left(\frac{1}{B}-1\right)} = \frac{M_T}{k\left(\frac{1}{B}-1\right)} = \frac{k_H \gamma \lambda_{sz} D^5 n_{sz}^2}{k\left(1-\frac{1}{B}\right)} \tag{193}$$

Since $n_{sz} = n_x$ therefore

$$M_x = \frac{1}{1-\frac{1}{B}} \frac{k_H}{k} \gamma \lambda_{sz} D^5 n_x^2 \tag{194}$$

and from this

$$\lambda_x = \frac{M_x}{\gamma D^5 n_x^2} = \frac{1}{1-\frac{1}{B}} \frac{k_H}{k} \lambda_{sz} \tag{195}$$

wherefrom

$$\nu = \frac{1}{1-\frac{1}{B}} \quad \frac{k_H}{k} \tag{196}$$

with the help of formulae (182), (187), (192) and (196), on base of the sample--charactertistics, the torque take-up characteristics of the hydromechanical stages were plotted (Fig. 109 - 112).

At all four types it can be unambigously stated that the torque take up varies dependent on B, according to a character nearly similar. Basic differences between the four types - not counted the shortening of the operational range - do not exist.

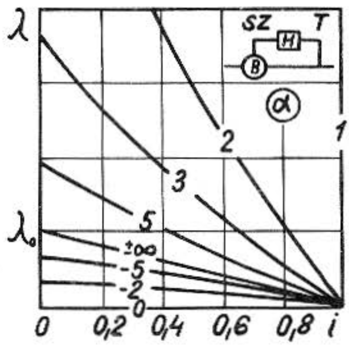


Fig. 109.

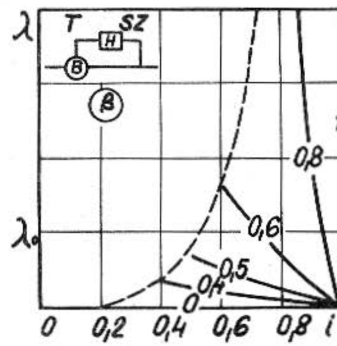


Fig. 110.

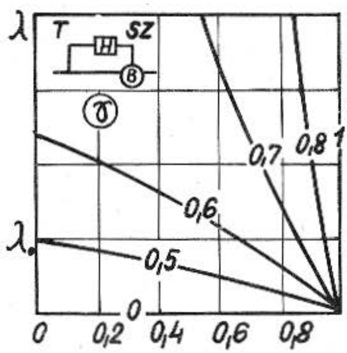


Fig. 111.

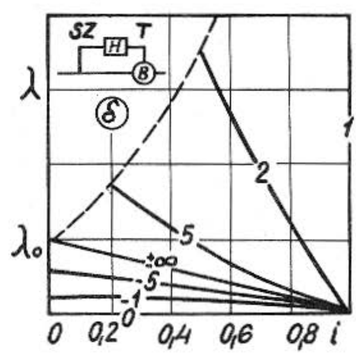


Fig. 112.

At all four increases the torque take-up when selecting a B value resulting in better efficiency.

The characteristics of torque take-up expediently are determined also in case of a clutch, resp. it is even necessary. These can be seen in Figs. 113 - - 116 for all four types, also on base of the sample-characteristics.

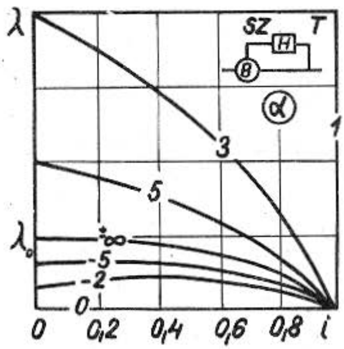


Fig. 113.

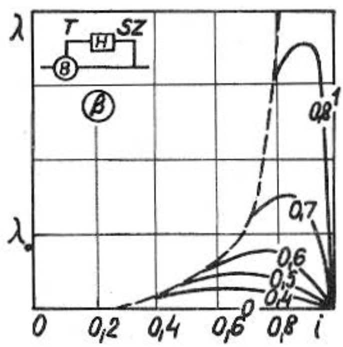


Fig. 114.

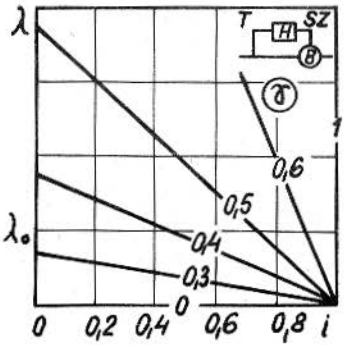


Fig. 115.

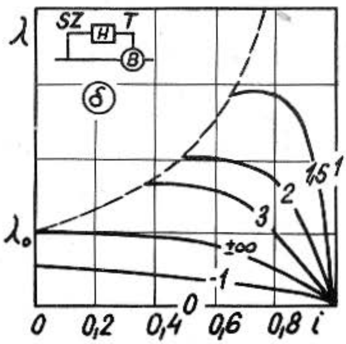


Fig. 116.

125. Amplified types. In our examinations up to this point we have taken as a base the simple case of the transfastened epicyclic gear when $i_o = i_\infty = i_{II} = 1$

In the following our examinations will be amplified and we will take in consideration also further connecting elements.

The effects of i_0 and i_∞ need no separate examination, since this was already performed at the simply fastened epicyclic gear. On the characteristics of the transfastened epicyclic gear, namely the i_0 and i_∞ transmissions have exactly the same effect as on the characteristics of the single hydraulic element. All which were said there, are valid also here without any change.

Before examining the effect of i_{II}, let us look at at the case, when the connecting element used for the transfastening of the epicyclic gear is not a single torque converter, but a torque converter and a mechanical transmission, in series connection (Fig. 117). Of the possibility of this, attention was called for already previously.

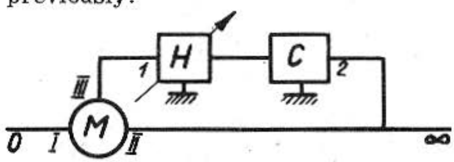


Fig. 117.

Let us mark the transmission series connected with the torque converter by c. In formulae (160 - 163) the i_c will be the factor of i_H in case of straight binding-in, while in case of reversed binding-in the divisor of i_H. In Fig. 118 we have plotted the efficiency curves of type α stage, in case of B = 2 at different i_c values. (Naturally, the curve pertaining to i_c = 1 can also be found in Fig. 105). As it can be seen, beside increasing i_c values the efficiency curve will be deviated to the right side, while the curve rises to the maximum. In case of decreasing i_c values the variation is the opposed one.

These same curves ought to be plotted also for other B values, in this way, however, we would get a figure rather hard to survey. So, instead of this, only the geometrical place of the maximum-points will be indicated. In Fig. 118 the dashed line marked by B = 2 already gives the place of the maximum-points, at optional i_c and B = c values. In Fig. 119 the same was extended also for an optional value B. This diagram illustrates the place, where the maximum of the efficiency curve would fall in case of different B and i_c values. If e.g. B = 2 and i_c = 0,5, then the maximum efficiency falls to point M, and to this line pertains the efficiency curve plotted by a dash- and dot line.

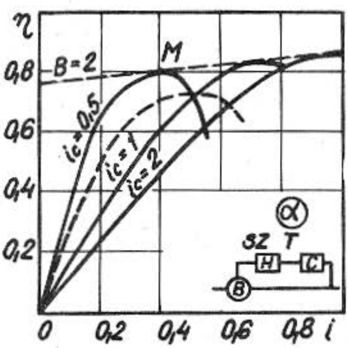


Fig. 118.

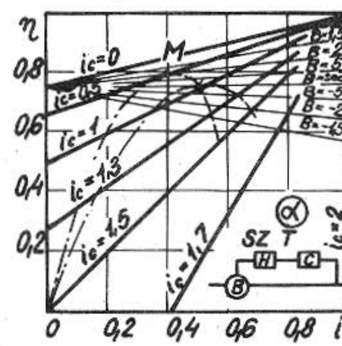


Fig. 119.

It can be seen, that by varying the i_c and B the efficiency curve - within certain limits - can be deviated to an optional place. (The original efficiency curve of the torque converter is indicated by a thin line.)

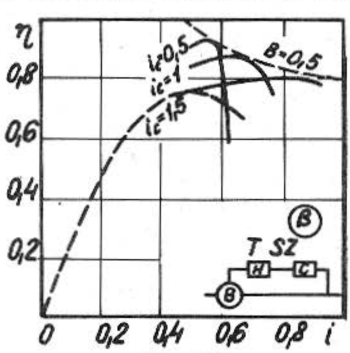


Fig. 120.

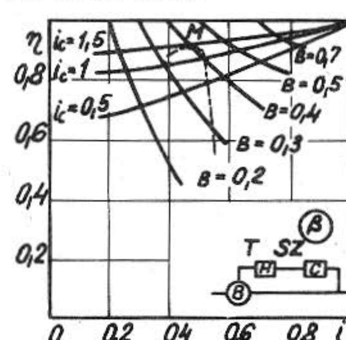


Fig. 121.

The curves of the efficiency-maximums were also plotted for the β, γ and δ type stages (Fig. 120 - 125). The diagrams of types β and δ show an especially advantageous picture. In both

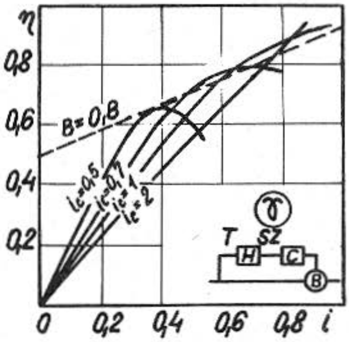


Fig. 122.

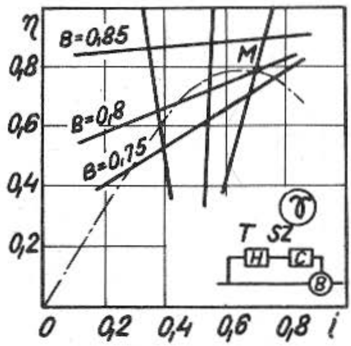


Fig. 123.

cases, namely, the maximum-point can easily be deviated to the left.

After all these, also the effect of transmission can easily be stated, Let us transform the formula of the kinematical transmission in case of one type, e.g. of δ .

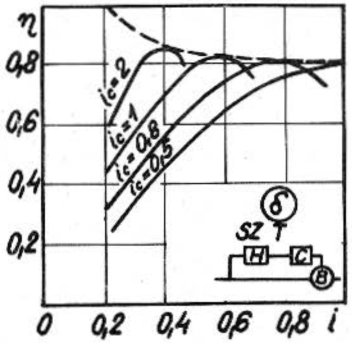


Fig. 124.

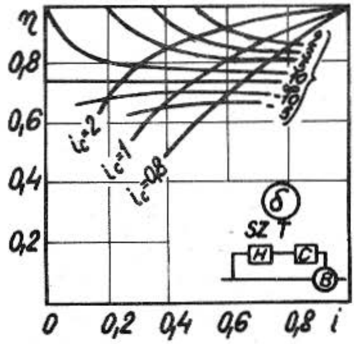


Fig. 125.

The formula of the kinematical transmission (66) for type δ is $i_{II} = 1$ but in case if $i_{II} = \dfrac{i_C}{i_H}$ it is the following:

$$i = \frac{\omega_y}{\omega_x} = \frac{\omega_o}{\omega_{\infty}} = \frac{1}{i_o\ i_{\infty}}\left[\frac{1}{B} + \left(1 - \frac{1}{B}\right)\frac{i_H}{i_C}\right] \tag{197}$$

while in case of $i_o = i_{\infty} = i_C = 1$

$$i = \frac{1}{B\ i_{II}} + \left(1 - \frac{1}{B}\right) i_H \tag{198}$$

The right side of the latter equation multiplied by $\frac{i_{II}}{i_{II}}$ is:

$$i = \frac{1}{i_{II}}\left[\frac{1}{B} + \left(1 - \frac{1}{B}\right) i_H \, i_{II}\right] \qquad (199).$$

Comparing equations (197) and (199) it can be seen, that $i_{II} = i_o \, i_\infty$ resp. $i_{II} = \frac{1}{i_C}$

$$i_o \, i_\infty = \frac{1}{i_C} \qquad (200)$$

which put into words means that a single transmission i_{II} has exactly the same effect as an $i_o \, i_\infty$ and an i_c transmission together, if between the latter two the relation (220) is valid. In other words, in the case when $i_o \, i_\infty = \frac{1}{i_C}$ then the two transmissions can be substituted by such a single transmission the value of which must be equal to $i_o \, i_\infty$

Application of i_{II} therefore, in certain cases makes the saving of a constant transmission possible. (This principle, otherwise, was patented by the Hungarian National Office of Patents [37].)

The effect of i_{II} can also be studied on the characteristics obtained up till now (119, 121, 123 and 125 figures), where the different i_C values are already indicated. The geometrical place of the efficiency maximums which will be found on curve $i_C = \frac{1}{i_{II}}$ in case of applying i_{II} will be only horizontally deviated and that in the proportion of the i_{II} In Fig. 126, for an example, for the $i_o \, i_\infty = 2$ case the efficiency characteristics of type δ can be plotted, which in the sense of the abovesaid was constrained in proportion of $i_o \, i_\infty = 2$ (See also Fig. 125!). In this Fig. the possibility $i_C = 2$ can be examined.

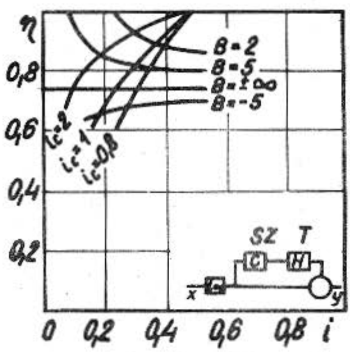


Fig. 126.

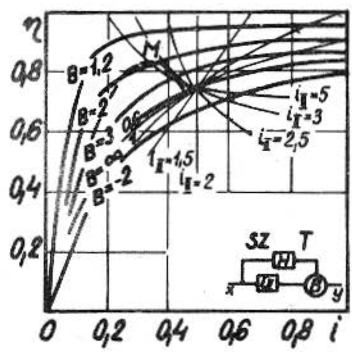


Fig. 127.

We find the efficiency maximum pertaining to it on the $i_C = \frac{1}{i_{II}} = 0{,}5$ curve, which in this case is a vertical straight. When taking the value of B, as an example, for 2, then the efficiency maximum $\eta_{max} = 0{,}87$ For the sake of illustration the characteristics showing directly the effect of i_{II} are separately plotted (Fig. 127). It can be seen that the efficiency $\eta_{max} = 0{,}75$ of the sample-

-characteristics which pertained to i = 0,5, by suitable selection of i_E and B, can be deviated to an optional place. As an example, the efficiency curve the maximum of which falls to point M and which was obtained in case of B = 2 and $i_{II} = 5$ was drawn-in.

13. The cross-fastened epicyclic gear as hydromechanical stage

The examination of the epicyclic gear cross-fastened by the hydraulic element, was performed only recently. The first reference to it was done in the papers 97, 99 of the author of this study, after this only in the short study of Seeliger do we find any trace of their existence.

For the examination of the epicyclic gear cross-fastened by the hydraulic element we will use likewise the relations deducted in Chapter II [99]. For plotting the kinematical characteristics we will start from formula (79):

$$i = \frac{\omega_\infty}{\omega_o} = \frac{1 - \frac{1}{B^N} + \frac{1}{B^N} i_H}{1 - \frac{1}{B^M} - \frac{1}{B^M} i_H} \tag{201}$$

which was obtained by substituting $i_{III} = i_H$.

As it can be seen, here are three independent variables as against the two independent variables of the transfastened epicyclic gear. Comparing it to the transfastened epicyclic gear, a further difference is, that here neither the reversed drive, nor the reversed binding - in of the hydraulic element has any significance, since the cross-fastened epicyclic gear - like the simply fastened one - is of simmetrical design.

Plotting of the kinematical characteristics - the graphic illustration of relation (201) - is because of the three independent variables somewhat difficult.

It seems to be the most simple solution to plot separate characteristics to each different M values, where the curves $i = f(i_H)$ pertaining to different B^N -s do not have a disturbing effect any more. For the sake of simplicity only six B^M values were taken: $B^M = (-0{,}5); (-5); (0{,}1); (0{,}5); (1); (5)$ These values were selected so that they should present characteristically the influence of B^M moreover, they should also well illustrate the cases pertaining to the critical B^M values (Figs. 128 - 133).

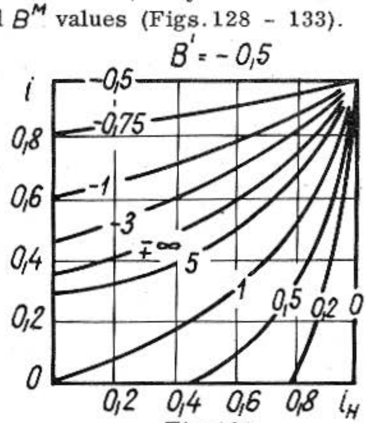


Fig. 128.

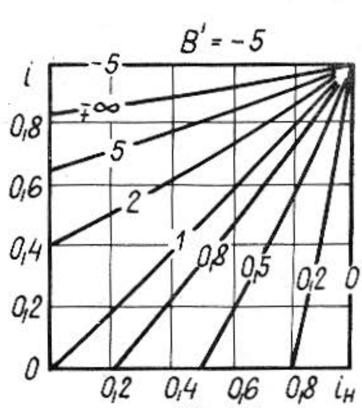


Fig. 129.

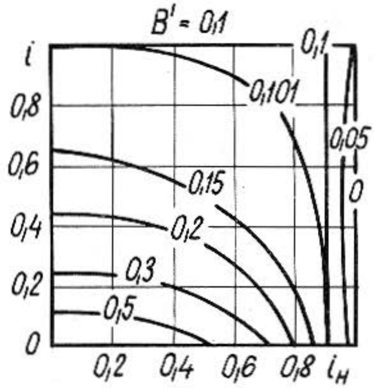


Fig. 130.

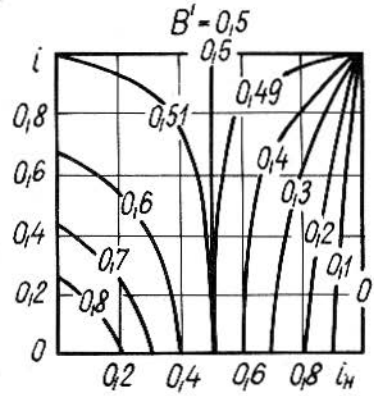


Fig. 131.

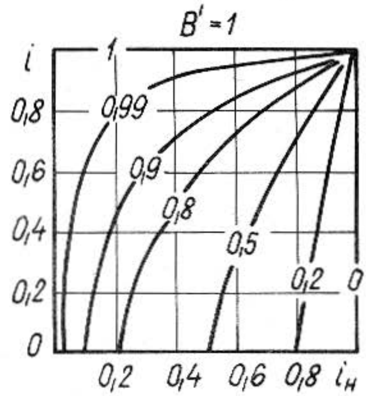


Fig. 132.

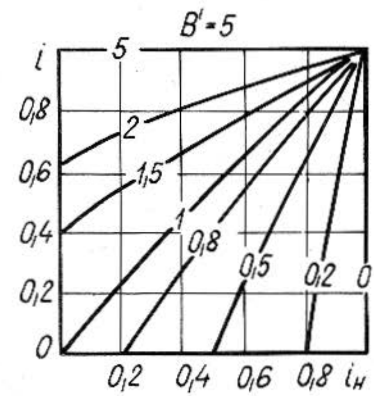


Fig. 133.

In Fig. 128 the curves pertaining to the $B^M = -5$ can be seen, which are near to the curves pertaining to the $B^M = -\infty$ which otherwise would be straight. The curves seen here, compared to the straight, are from below somewhat convex. When comparing this Fig. 128 to Fig. 100, the similarity will be apparent, moreover, if $B^M = -\infty$ is taken in consideration, then the characteristics are in their shape completely conform. The difference is merely as much that to the same straight not the same B^N values pertain (e.g. in Fig. 100 the oblique straight of 45° pertains to the $B = \pm\infty$, while in case of the crossfastened epicyclic gear to the $B = 1$). This means that the crossfastened epicyclic gear in case of $B^M = -\infty$ will be transformed to a transfastened epicyclic gear of the type δ with the difference that the shafts II and III of the epicyclic gear B^N are changed.

This transformation happens therefore, because shaft II of the epicyclic gear M in case of $B^M = -\infty$ ceases to function, i.e. shafts I and III of the epicyclic gear become in rigid connection with each other. The difference being in parameter B^N occurs because of the shaft-change, that, however, does not change the essence.

In Fig.129 merely as much change can be seen compared to Fig.128 that the convex shape of the curves become more prevailing, which - continuing the former thought - means that the characteristics of the crossfastened epicyclic gear along with the nearing of the B^M from $-\infty$ to o, will be ever less similar to the characteristics of the transfastened epicyclic gear of the type δ

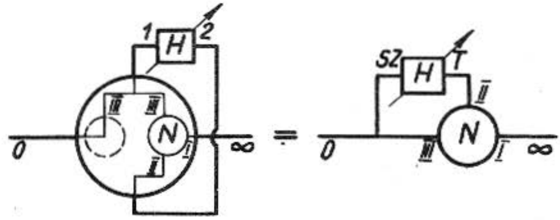


Fig.134.

When the value of the B^M rises over O, the character of the curves will be changed. In Figs.130 and 131 the character of the curves is well discernible: at $i_H - 1 - B^M$ a vertical straight will be obtained which, otherwise, pertains to $B^N = B^M$ From the right and left of this straight a set of curves with opposed curvatures. are seen. The set of curves on the left side along with the increase of the B^M will ever more become similar to Fig.99, while in case of $B^M = 1$ (Fig.132) it is already identical to it. This means that the crossfastened epicyclic gear then will be transformed again to a transfastened one of the type γ, since now the shafts I and II of the epicyclic gear M are rotating together (Fig.135). By increasing further the B^M the curves begin to be straightened again and to get similar to Fig.128, only with an opposed curvature (Fig.133). This means that at $B^M = \infty$ we arrive again to the backfastened epicyclic gear of type δ

When examining the critical B^N values, similar conclusions can be drawn, but this time we may obtain prefastened epicyclic gears. Let us e.g. look at the curves pertaining to $B^N = 1$ in Figs.128, 129 and 133. As can be seen, these may be found together also in Fig.97 where to the different values B the values B^N found here do correspond. In case of $B^N = 1$ therefore, the crossfastened epicyclic gear will be transformed to one prefastened and of the type α (Fig. 136). In Fig.98 also the curves pertaining to $B^M = \pm\infty$ in the Figs.128 and 129 can be found, though here the values B do not correspond to the values B^M because of the shaft-change (Fig.137). Independently of this in case of $B^M = \pm\infty$ the crossfatened epicyclic gear will be transformed to a prefastened one of type β

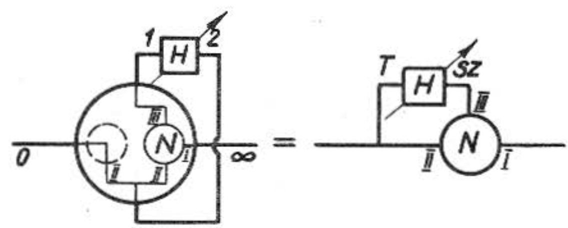


Fig.135.

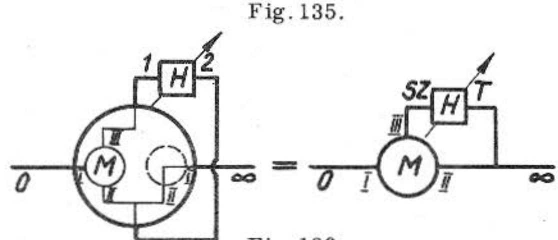


Fig.136.

Summing up the above, it can be stated that <u>the corssfastened epicyclic gear is no-</u>

thing but the amalgamation of the transfastened epicyclic gears of the types α, β, γ and δ as components.

The composition, i.e. the proportion of part-taking of the individual components depends on the value of B^M and B^N The characteristics of the crossfastened epicyclic gear can be shaped optionally by varying the value of the two B-s and certain properties of the characteristics of the individual components can be prevailing or pushed into the background. When necessary, any one of the components can be produced also alone. This gives a new understanding also to the transfastened epicyclic gears, since they, according to the above, can be considered as special cases of the crossfastened epicyclic gear (naturally, also in case of purely mechanical connecting elements!).

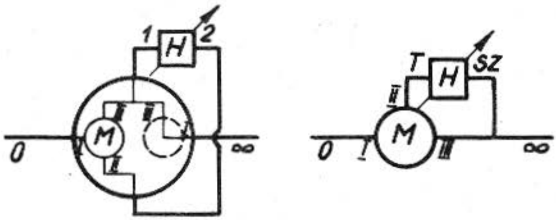


Fig. 137.

After all these, let us, as an example, examine in some cases the variations of the torque transmission and efficien-

In Figs. 138 and 139, in case of $B^M=5$ the two characteristics are seen. They are characterized by the great extent of the shortening of the operational range in case of more adventageous efficiency as it otherwise follows also from the kinematical characteristics (Fig. 128). For Figs. 140 and 141 the advantageous torque transmission resp. efficiency is characteristic which occurs around $B^M=-0,5$. It is experienced already here, but at $B^M=5$ (Figs. 142 and 143) it presents itself all the more that the curves do not run along the complete operational range as it would follow from the kinematical characteristics (Fig. 138).

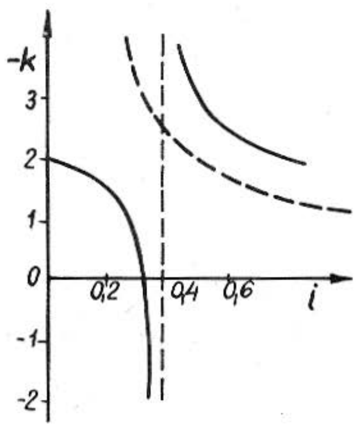


Fig. 144.

The cause of this is that the torque transmission resp. the efficiency curves become over a certain value i irrational.

Let us examine e.g. the curve $B^N=0,2$ of Fig. 142 apart (Fig. 144). The curve starts from point i = O, k = 2, at approx. the i = = 0,3 goes through the basic line and approximates the vertical assimptote at cca i = = 0,4. To the right of the assimptote the curve again is continued in the upper part, above the ideal torque converter hyperbola. To such torque transmission would pertain, of course, an efficiency better than $\eta = 1$.

When, however, the direction of the performance-flow is examined (dispensing with the deduction), it will be clear, that in this operational range the hydraulic element is in rational service: though the kinematical transmission falls between O and 1 (in the operational range), moreover, the torque trans-

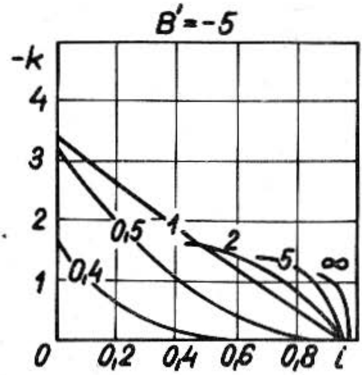


Fig. 138.

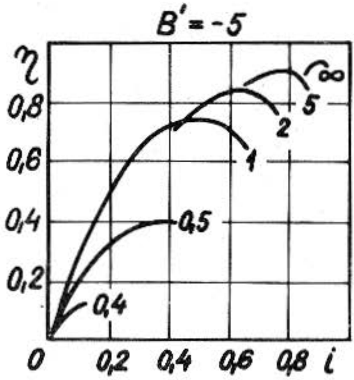


Fig. 139.

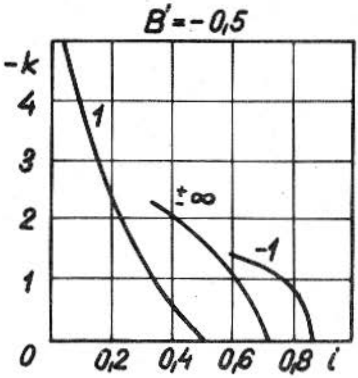


Fig. 140.

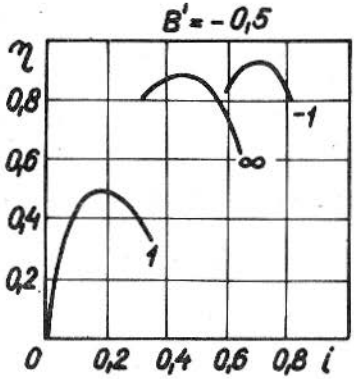


Fig. 141.

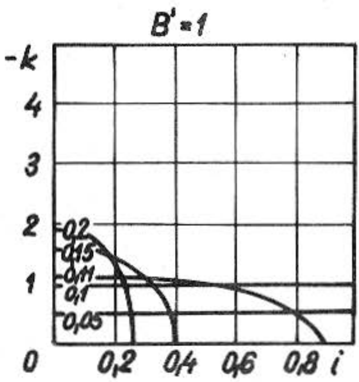


Fig. 142.

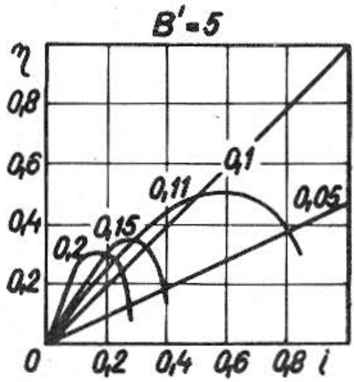


Fig. 143.

mission k_H also corresponds to characteristics $k_H = f(i_H)$ i.e. the k_H falls between (-s) and O, this negative value is, however, obtained not so that the torque of negative sign acting on the turbine wheel will be divided by the torque of positive sign acting on the pump impeller, but in the reversed manner. The change of sign has no effect upon the sign of the quotient, but it reverts the operation of the hydraulic element: the performance comes-in on the turbine wheel, inside more performance will be added to it, then the increased performance goes out on the pump impeller. The hydraulic connecting element, of course, is unsuitable for such role, just therefore, in Figs. 138-143 the irrational sections of the curves were not plotted.

2. DESIGN OF HYDROMECHANICAL CHANGE-SPEED GEARS IN PRINCIPLE

We talk of hydromechanical change-speed gear then, when among its stages at least one hydromechanical stage can be found.

There is, therefore, for designing a hydromechanical change-speed gear only one way: the selection of one of the stages described in the foregoing paragraph. (The socalled direct stage is also here not counted as a separate stage.) So the theoretical design of the change-speed gear of one stage can be of three kinds, depending on that, whether a simply fastened, transfastened or crossfastened epicyclic gear will be selected. In connection of the application of the simply fastened epicyclic gear it is worth-while to mention an interesting variety. In Fig. 145 we can see such a hydraulic element which has two outgoing shafts independent of each other. Into one of the outgoing shafts a simply fastened epicyclic gear will be inserted, the two shafts will be unified. Naturally, also into the other outgoing shaft can a simply fastened epicyclic gear be inserted, moreover, one more after the uniting of the two shafts. There can be imagined such a hydraulic element, which has three or more outgoing shafts, so the design of the change-speed gear may be processed still further.

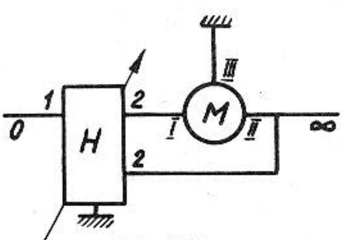


Fig. 145.

The change-speed gear of two stages has two basic types:

a/ one hydromechanical stage + one mechanical stage (HM+M)
b/ two hydromechanical stages (HM+HM)

As the mechanical stage of the first type, any stage dealt with in Chapter III may come in consideration, for the hydromechanical stage the abovesaid are valid.

At the second type the two hydromechanical stages can be of identical or of different design.

The formula of the change speed gear of three stages is according to the foregoing:

a/ HM + M + M
b/ HM + HM + M
c/ HM + HM + HM.

The number of stages can be, of course, further increased, only that must be cared for that the hydromechanical stages should always be the commencing ones. The selection of the type of the hydromechanical stage is possible only after the critical examination of them was performed. This critical examination, however, exceeds the limits of this study, we should like only to call the attention to some viewpoints. It can be stated generally that the different species and types of the hydromechanical stages have different advantages and disadvantages, so every one of them can be used maximally for one or two purposes. When remaining at the motor car drive then it can be stated that e.g.

a/ it is not expedient to apply a transfastened epicyclic gear as the starting stage of the change-speed gear, since along with the increasing of the torque modification the efficiency decreases. For this purpose is the simply fastened epicyclic gear the most suitable, where the mechanical and hydraulic elements are series connected, because there the efficiency will not change:

b/ as a speeding-up stage the simple torque converter is the most adequate or eventually the transfastened epicyclic gear of type β or δ :

c/ as a final stage it is expedient to use the transfastened epicyclic gear of the type δ perhaps that of type β :

d/ the stages of the type α and γ are not suitable for use in the change speed gear.

To illustrate the above two sample-characteristics are presented.

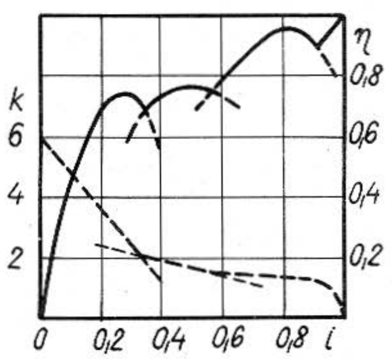


Fig. 146.

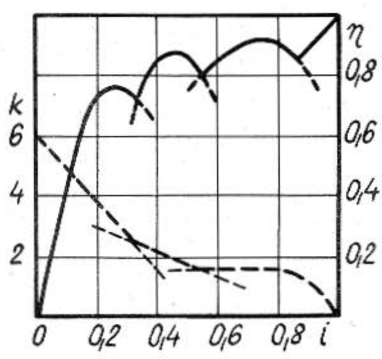


Fig. 147.

In Fig. 146 the three stages are the following:

Stage I: series connected B + 0,5 epicyclic gear
Stage II: torque converter, alone
Stage III: transfastened epicyclic gear of type δ (B = 1,5).

In Fig. 147:

Stage I: series connected B = 0,5 epicyclic gear
Stage II: transfastened epicyclic gear of type β (B = 0,35)
Stage III: transfastened epicyclic gear of type δ (B = 2).

In both Figs. the change of the modification factor was also plotted.

It must be noted that in practice torque converters of much better efficiencies as the sample-characteristics are produced, so that in the reality much more advantageous results may be obtained by the epicyclic gear combinations.

The advantageous result will, however, appear only then, if we use few epicyclic gears, because in the reality also the efficiency of the epicyclic gear must be counted with.

V. EXAMPLES OF CHANGE SPEED GEARS WITH EPICYCLIC GEARS

1. MECHANICAL CHANGE SPEED GEARS

As can be seen from the foregoing, incalculably various cases of epicyclic gears may be possible, just therefore was the analysis needed. As a result of our researches, any epicyclic gear existent or to be produced only afterwards can be examined by an uniform method, its place in the system can exaxtly be determined and its properties outlined beforehand.

We can not offer here the illustration of all existent or up till now designed change-speed gears with epicyclic gear, we can present only a few, more known examples, selected at random.

In Fig. 148 an old change-speed gear can be seen [102] which is also of constant inside design. Accordingly, in the change-speed gear four simple epicyclic gears can be found. This generally known Wilson change-speed gear corresponds to the type 3 of Fig. 72. The Wilson change-speed gear of five stages also was plotted (Fig. 150) which is a further developed variation of the former; it got one more stage prefastened by a simply fastened epicyclic gear.

As a deterrent example also the change-speed gear of four stages consisting of a severalfold-complex epicyclic gear is shown in Fig. 151. [74] the basic fault of which is that it has too many geared wheels, parttaking in the drive, in all stages.

The two following examples show the beginnings of the change-speed gear of varying inside design. In the change-speed gear of three stages of Fig. 152-[75] only one epicyclic gear is found. The two clutches on the incoming shaft give possibility for certain permutation. The first two stages still are identical to the third basic type of Fig. 66, while the third stage is the result of the changed binding-in of the epicyclic gear M.

In Fig. 153 the Hobls "Mechamatic" change-speed gear is presented. At this the fourth stage is obtained by changing the binding-in of the formerly already utilized epicyclic gear O. The first three stages otherwise correspond to the 8^{th}-type of Fig. 72, with reversed drive.

A tipycal example for the change-speed gear with varying inside design is the change-speed gear plotted at the motor vehicle faculty of this University [100]. As it may be seen in Fig. 154, this construction can have 18 stages in all.

The selection of the needed six stages and the determination of the values B pertaining to it can be performed most simply by the help of an electronic computer.

The Cotal change-speed gear was already mentioned in the chapter about the change speed gear of two members 102 . In Fig. 155 can be seen that this structure is, after all, a change-speed gear of three members. The first three stages are obtained by the variation of the two last stages, while the reverse gear is secured by the instep of the first member. As a matter of fact, for the reverse gear we obtain also three stages but of them only the first is used, the other two were not plotted at all.

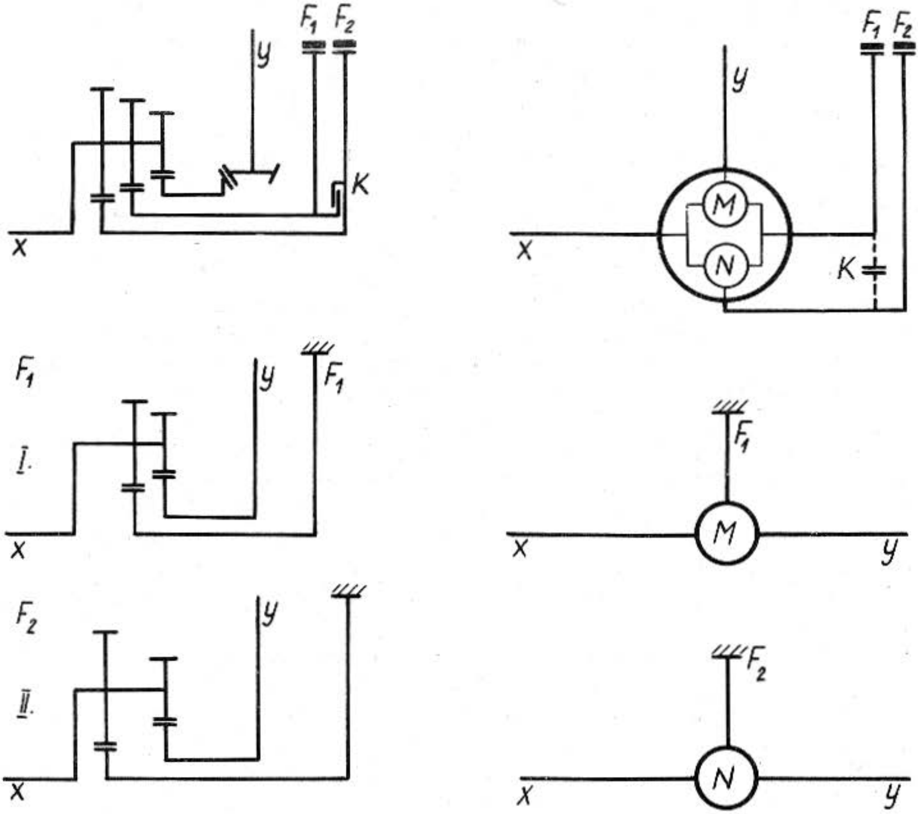


Fig. 148.

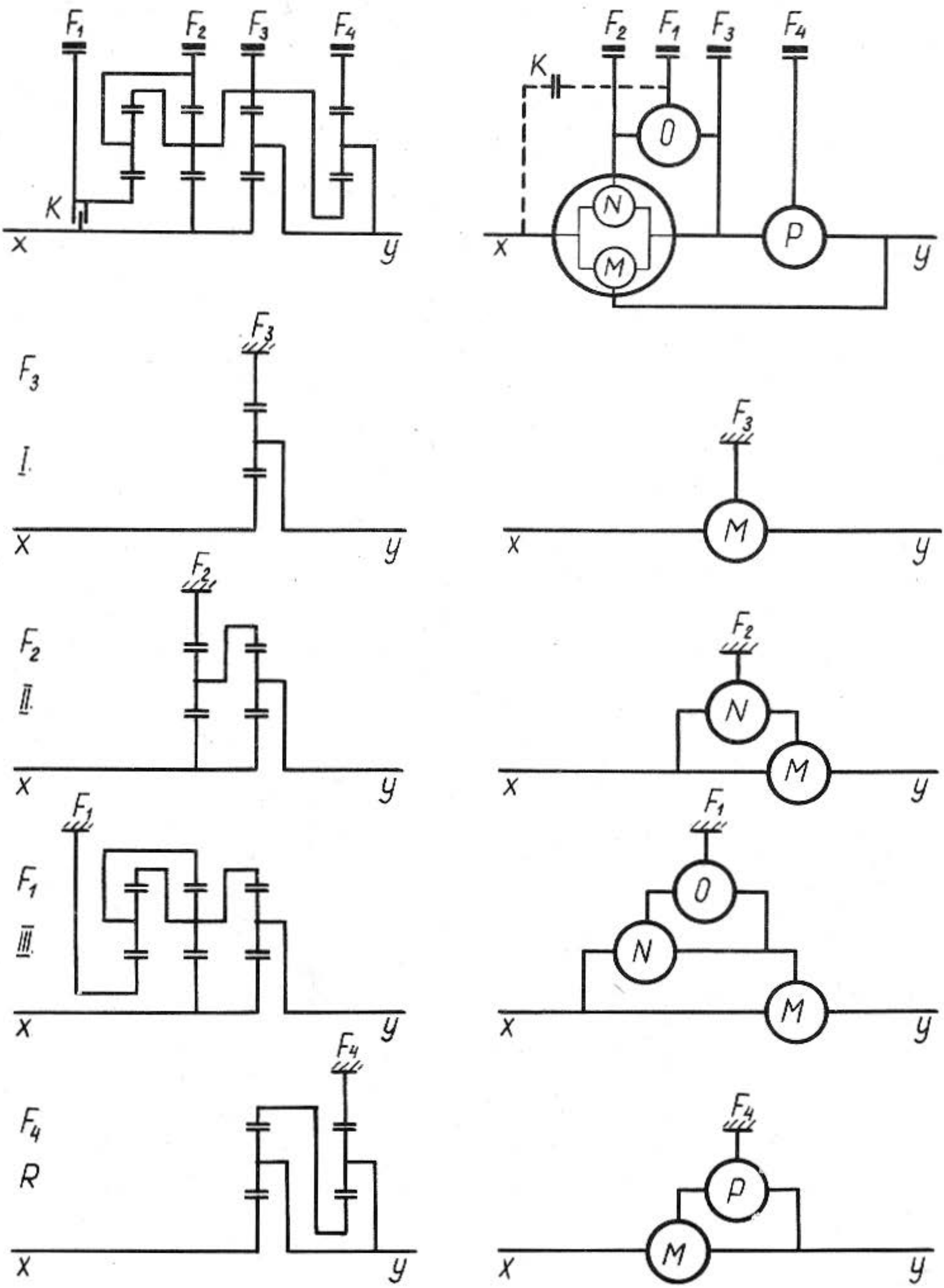


Fig. 149

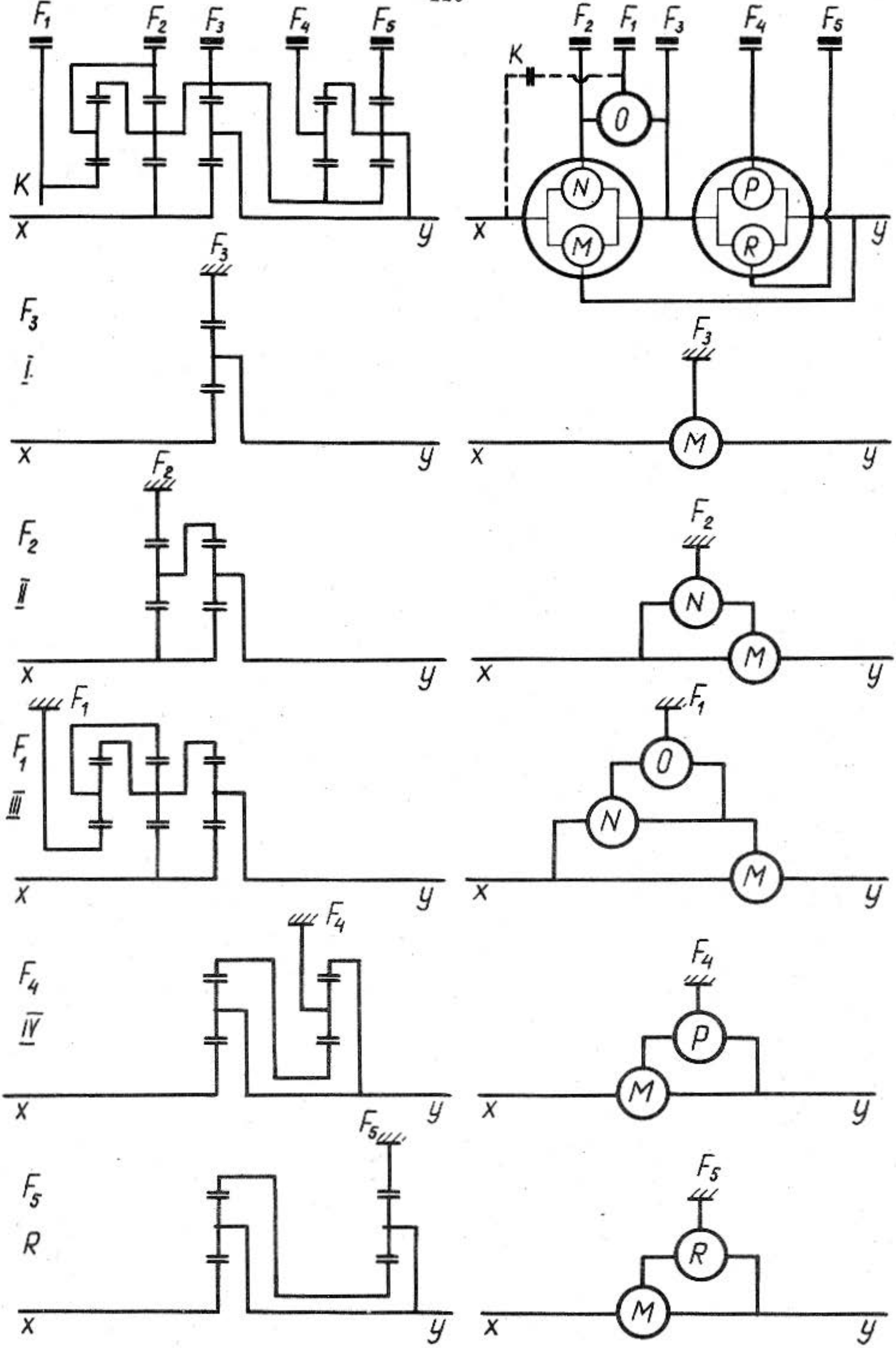


Fig. 150

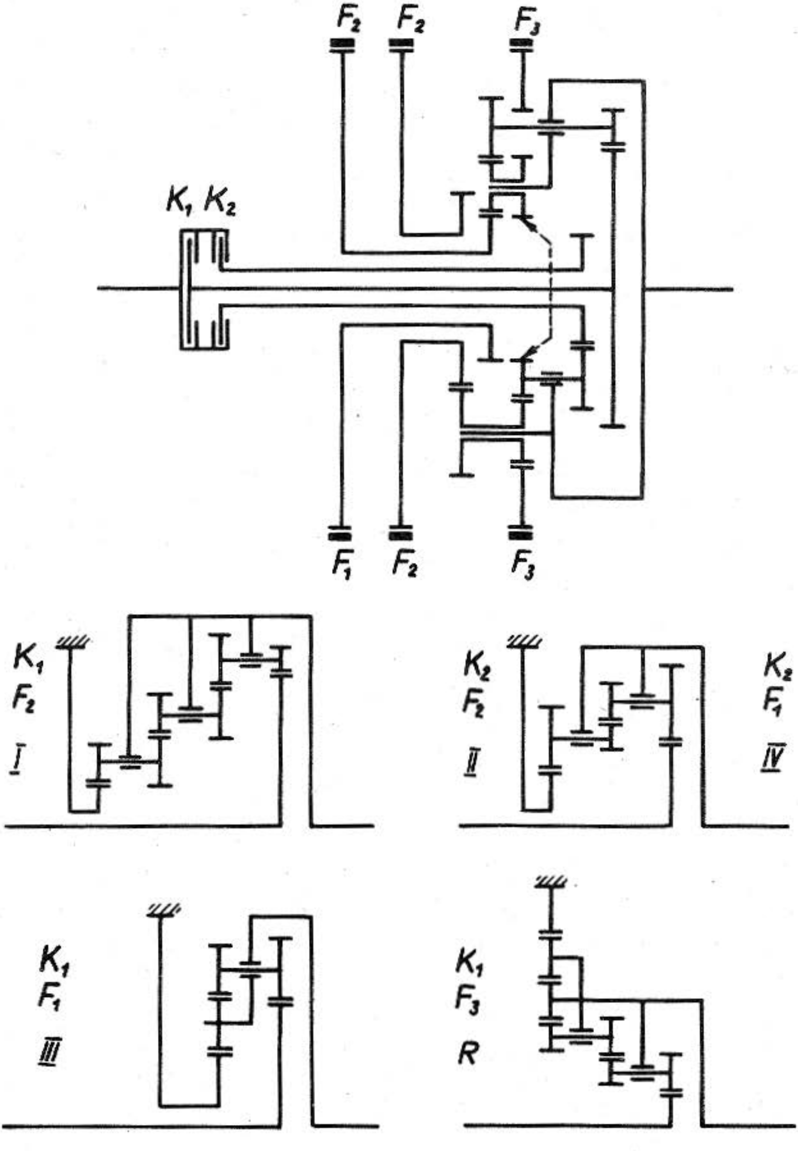
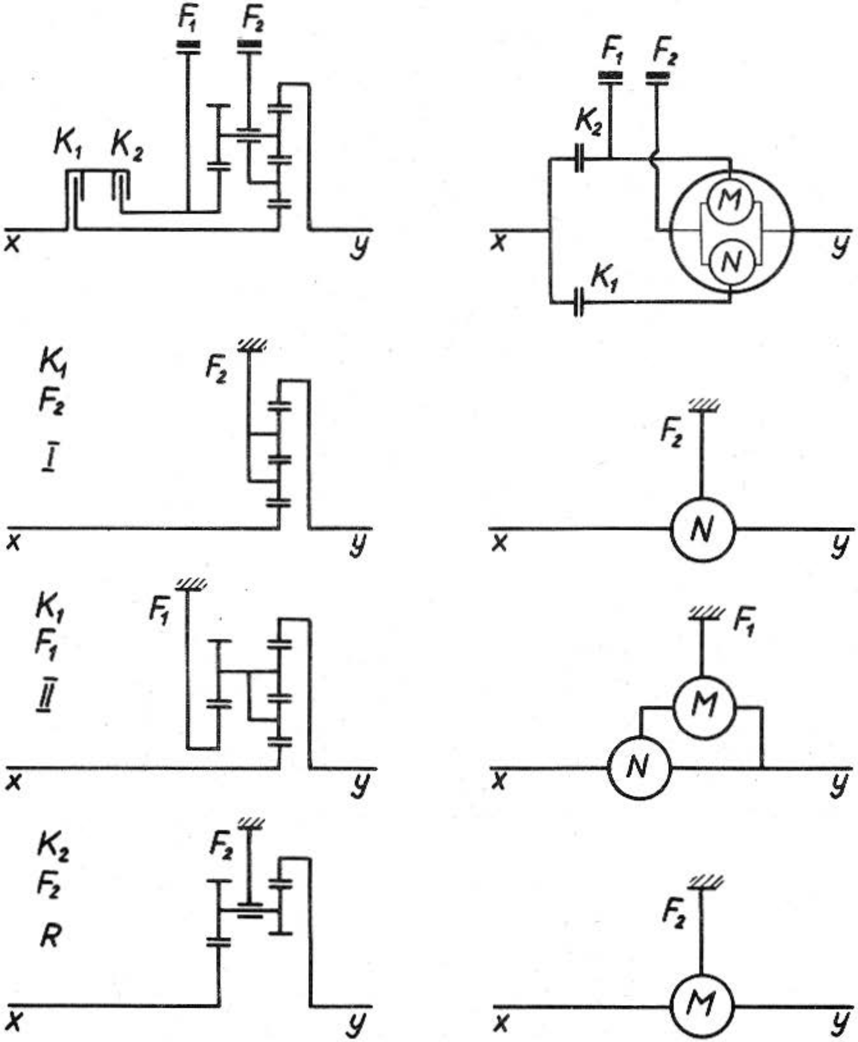
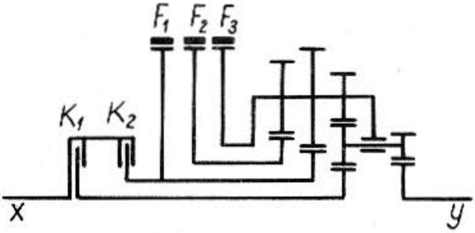


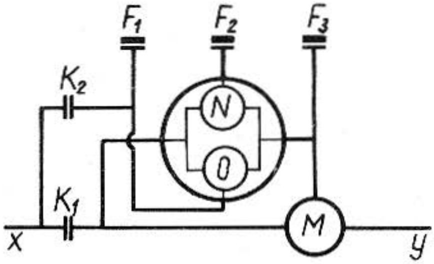
Fig. 151



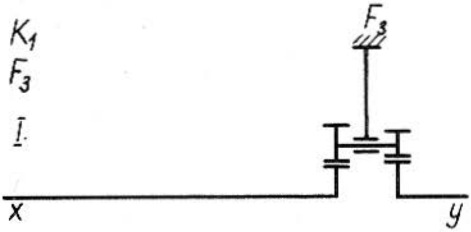
F1 F2
K1 K2
x
y
F1 F2
K2
M
N
K1
x
y
K1
F2
I
F2
x
y
F2
N
x
y
K1
F1
II
F1
x
y
F1
M
N
x
y
K2
F2
R
F2
x
y
F2
M
x
y

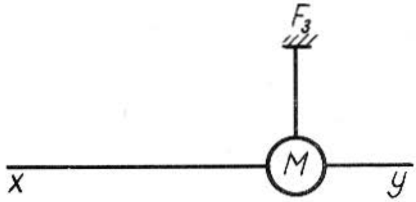
Fig. 152



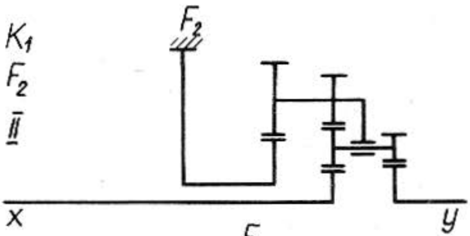


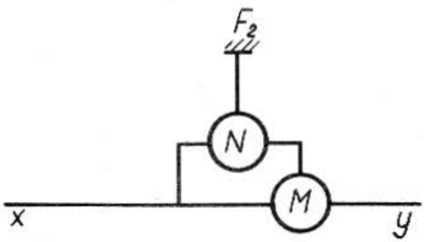
K_1
F_3
I



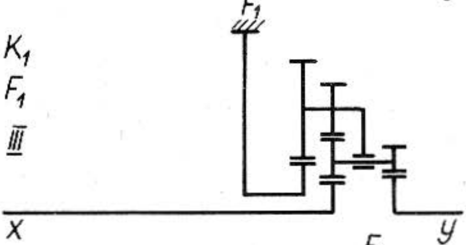


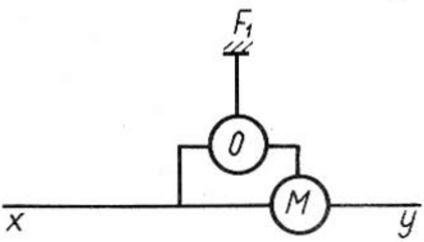
K_1
F_2
II



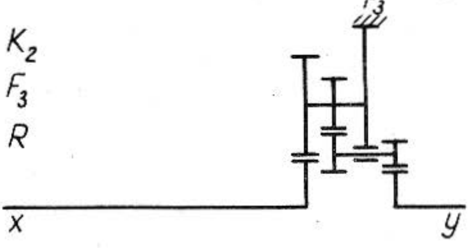


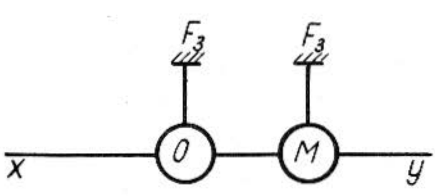
K_1
F_1
III





K_2
F_3
R





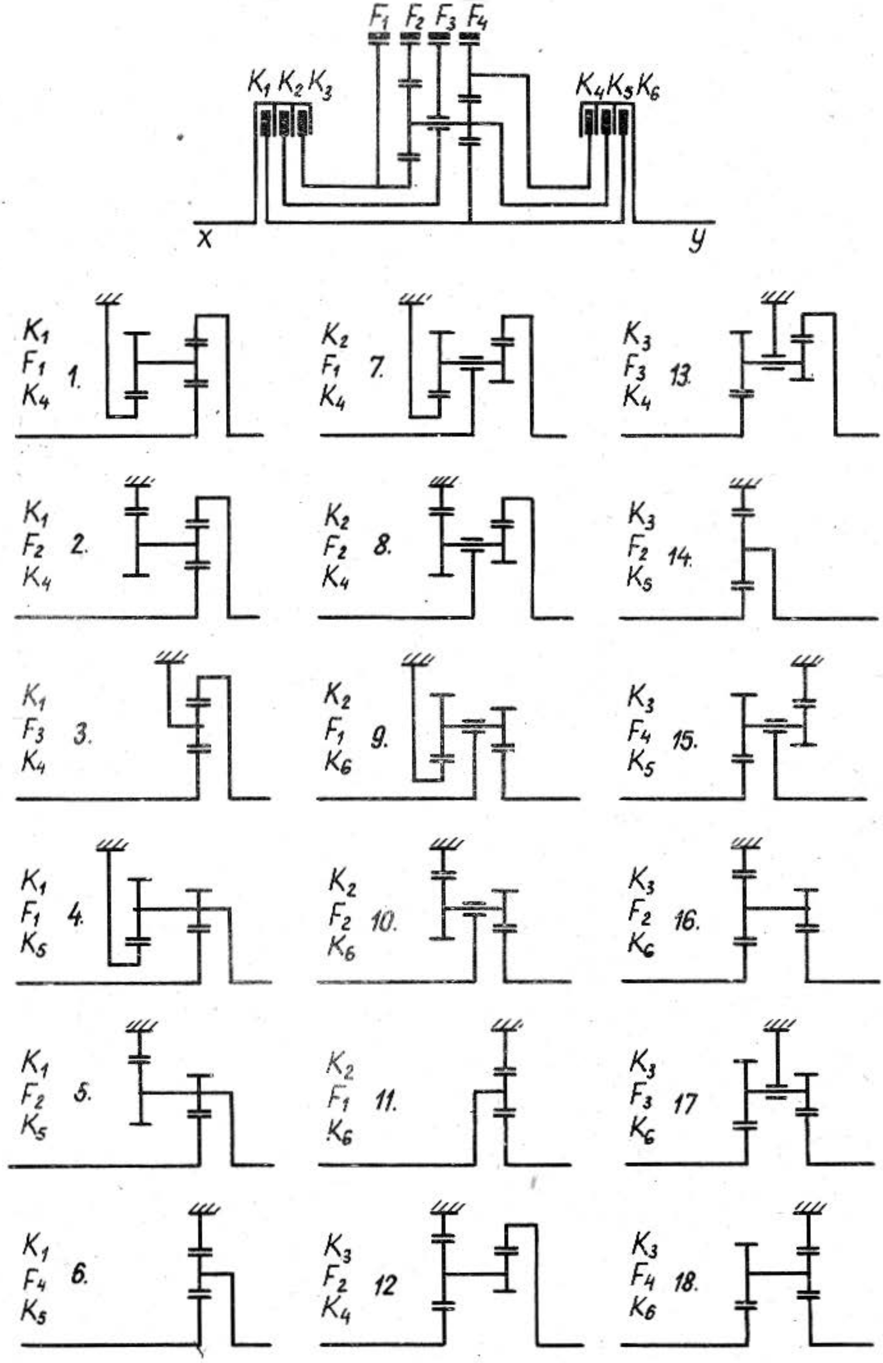

F1 F2 F3 F4
K1 K2 K3
K4 K5 K6
x
y
K1 F1 K4 1.
K1 F2 K4 2.
K1 F3 K4 3.
K1 F1 K5 4.
K1 F2 K5 5.
K1 F4 K5 6.
K2 F1 K4 7.
K2 F2 K4 8.
K2 F1 K6 9.
K2 F2 K6 10.
K2 F1 K6 11.
K3 F2 K4 12
K3 F3 K4 13.
K3 F2 K5 14.
K3 F4 K5 15.
K3 F2 K6 16.
K3 F3 K6 17
K3 F4 K6 18.

Fig. 154

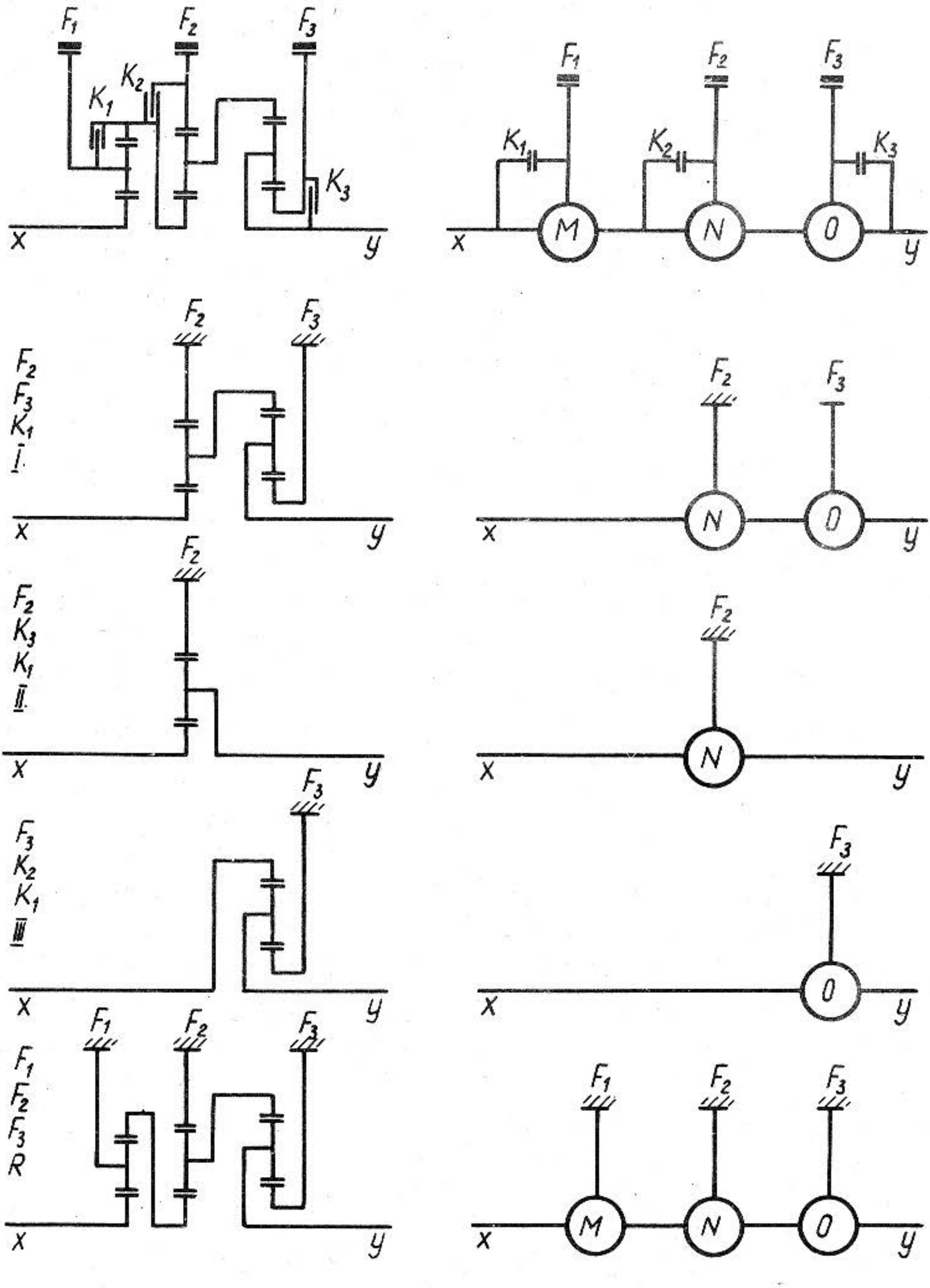


Fig. 155

2. HYDROMECHANICAL CHANGE-SPEED GEARS

As already mentioned, in the most simple type of hydromechanical change-speed gear simply fastened complex epicyclic gear is applied, where then hydraulic connecting element is inserted into the incoming or outgoing shaft, i.e. the hydraulic and mechanical elements are series connected. A further developed variation of this is, when the mechanical element is a change-speed gear of more stages which may contain not only simply fastened epicyclic gears, but essentially any type of the mechanical change-speed gears described in the previous paragraph can come in consideration.

In Fig.156 the "Dynaflow" driving gear 103 is shown, where the mechanical part corresponds to the 2 basic type of Fig.66.

The Kreisler Zil 111 change-speed gear was produced by the series connection of type 3 of Fig.66 and a hydrodinamical torque converter. (Fig.157) 103 .

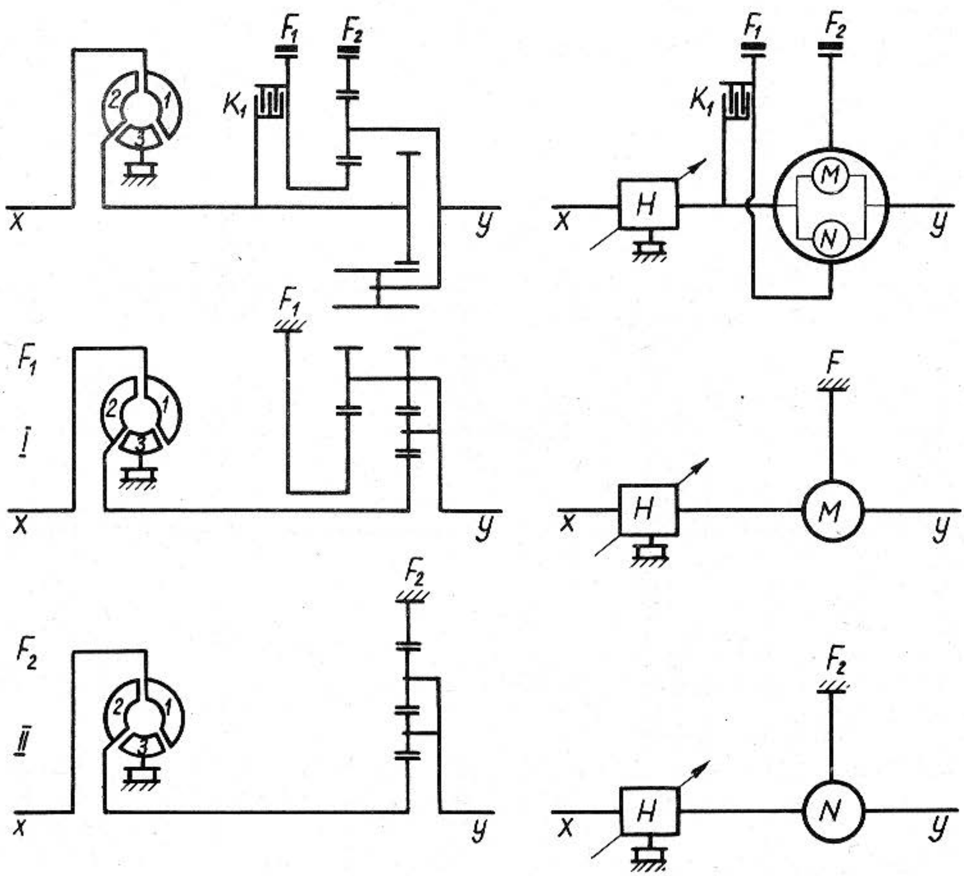


Fig.156.

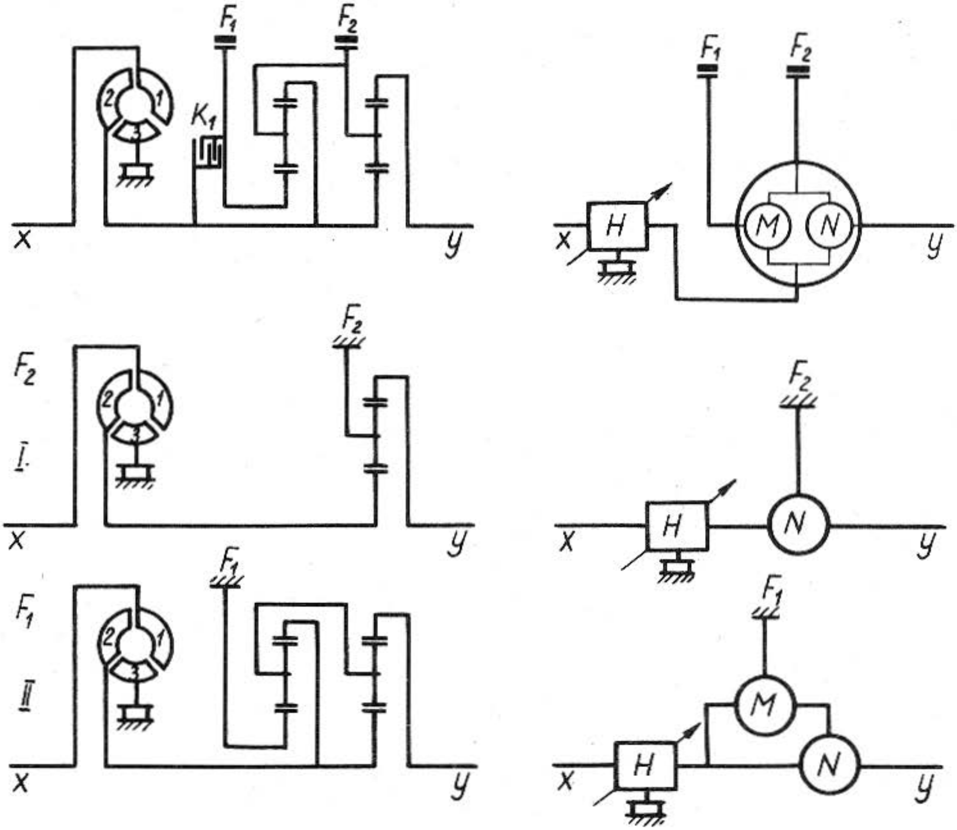


Fig. 157.

A very interesting application of the complex epicyclic gear may be met with at the Studebaker type of 1950 [69], where outside of the normal utilization of the complex epicyclic gear consisting of two simple ones (the first and third stages correspond to basic type 2 of the third stage) by shortclosing of the one of the simple epicyclic gears a third stage is obtained (Fig. 158). Because of the possibility of shortclosing, the shaft III of the two simple epicyclic gears are not definitely connected within the epicyclic gear, but by a clutch K_2 outside of it, which, when necessary, can be released.

For shortclosing, the clutch K_2 is released, and by clutch K_1 the opened complex epicyclic gear will be crossfastened (second stage).

The same principle may be seen in Fig. 159 too [89], where, however, the complex epicyclic gear, which can be opened, was used for transfastening.

The beginnings of the change-speed gear with epicyclic gear and of varying inside design can also be found in hydromechanical variation e.g. at the families "Torque-Flite" 44 and "Fordomatic" [103].

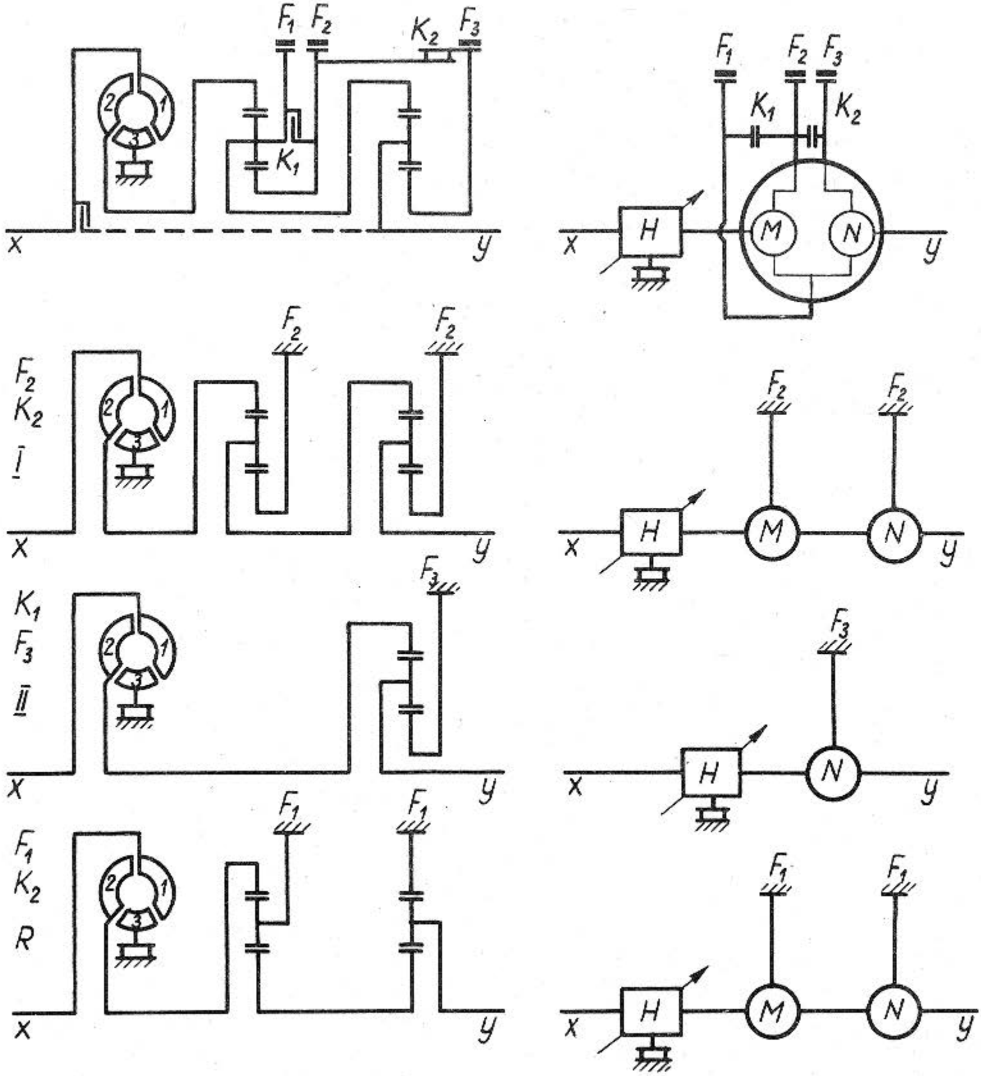


Fig. 158.

In Fig. 160 (Torque-Flite) the basic type (the first two stage) is the 3. variation of Fig. 66.

The third stage was obtained by the changed shaft-binding-in of the epicyclic gear M.

In Fig. 161 (Fordomatic 1949, Mercomatic 1951, Cruise-O-Matic 1958, Turbodrive 1955, Volgomatic 1956, Csajka 1960, etc. etc.) essentially a change speed gear of the same theoretical design can be seen, only here the complex epicyclic gear is at the same time a conjunct one.

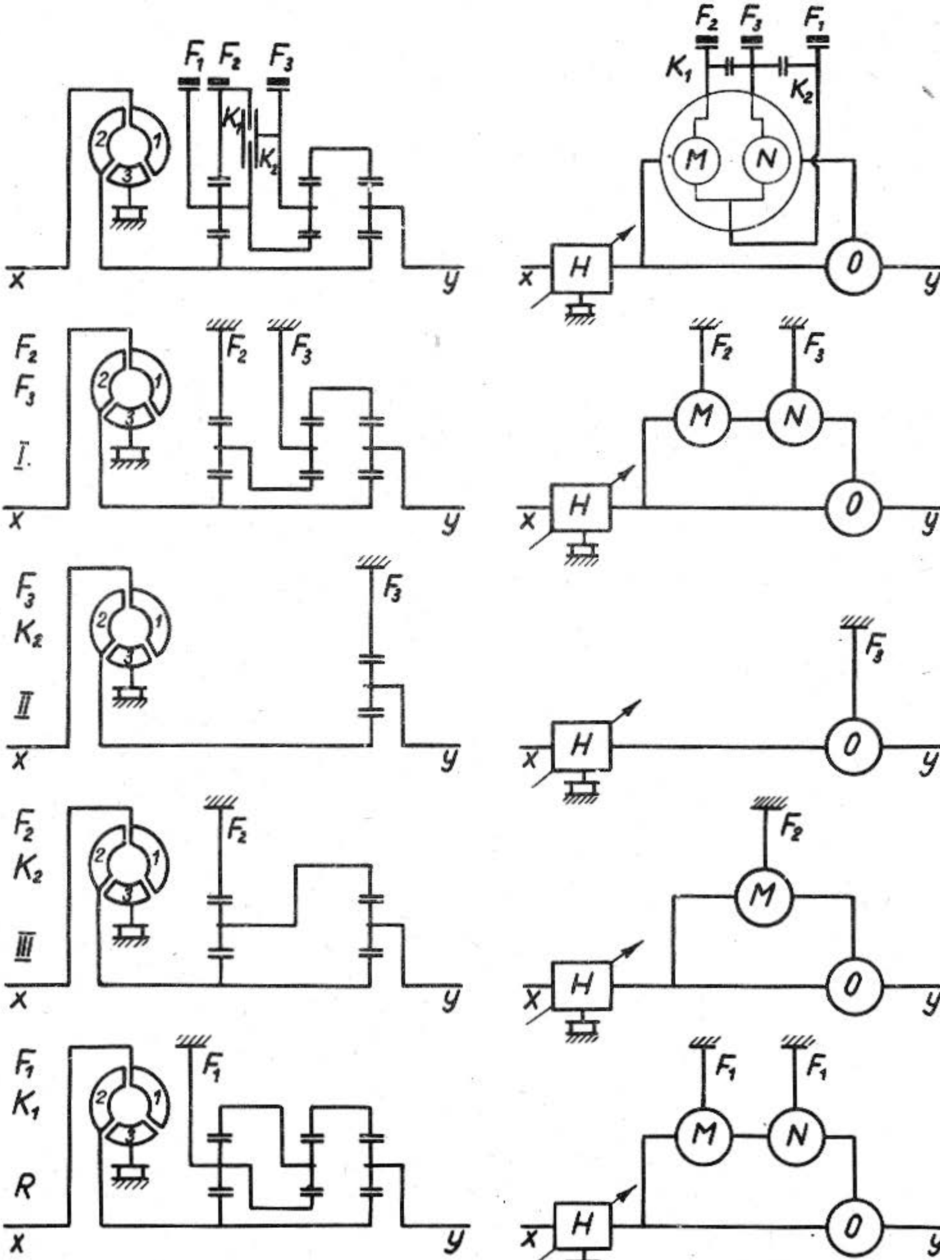

F1 F2 F3
K1 K2
1 2 3
x y
F2 F3 K1 F1
K1 K2
M N
H O
x y
F2
F3
I
F2 F3
M N
H O
F3
K2
II
F3
H O
F2
K2
III
F2
M
H O
F1
K1
R
F1
F1 F1
M N
H O

Fig. 159

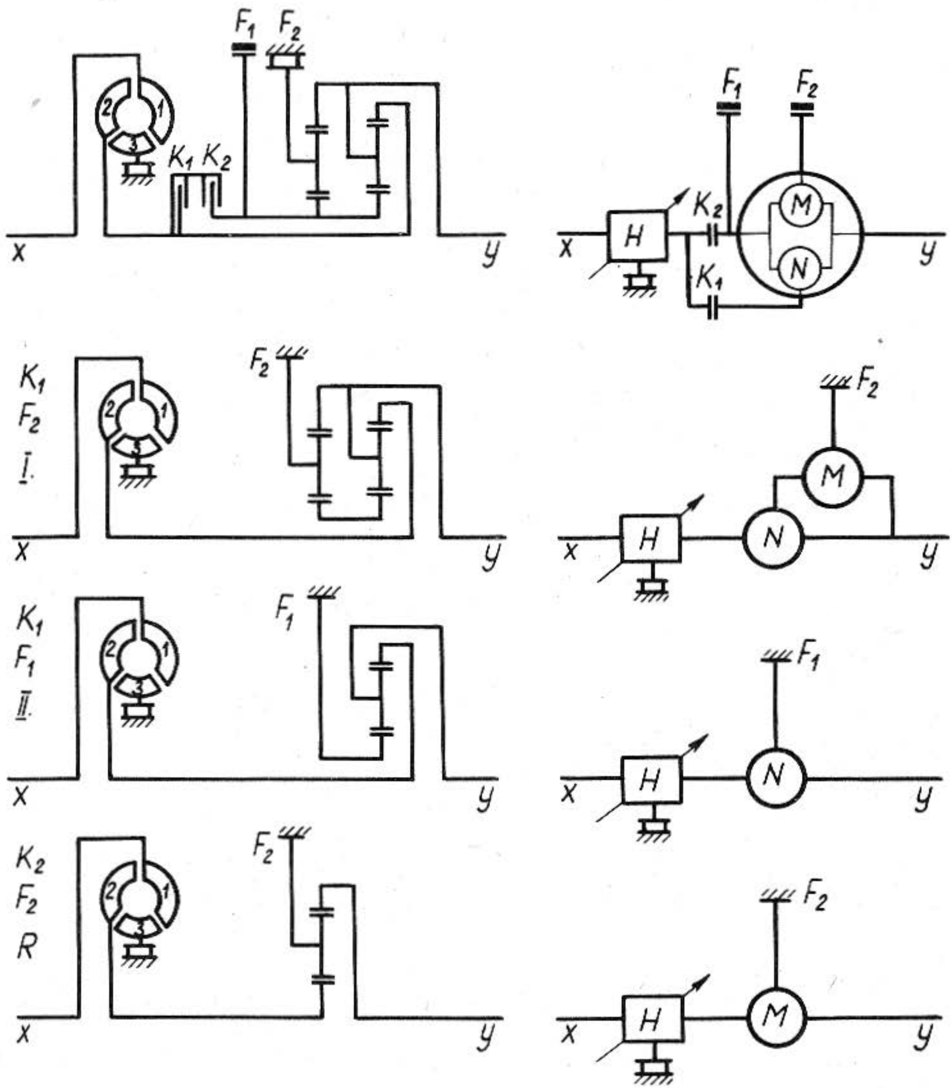


Fig. 160.

Fig. 162 pertains also to this group, it is, however, interesting because in the conjunct epicyclic gear bevel-gearing was applied and the permutation is performed by the outgoing shaft [73].

A beautiful example of the change-speed gear of two members is the Powermatic [55] driving gear which has outside of the "direct" seven stages (Fig. 163). One member of the driving gear is an epicyclic gear which can be simply fastened resp. shortclosed, its other member is a change-speed gear of three stages according to basic type 2 of Fig. 72 which can also be shortclosed. The seven stages are produced according to the following.

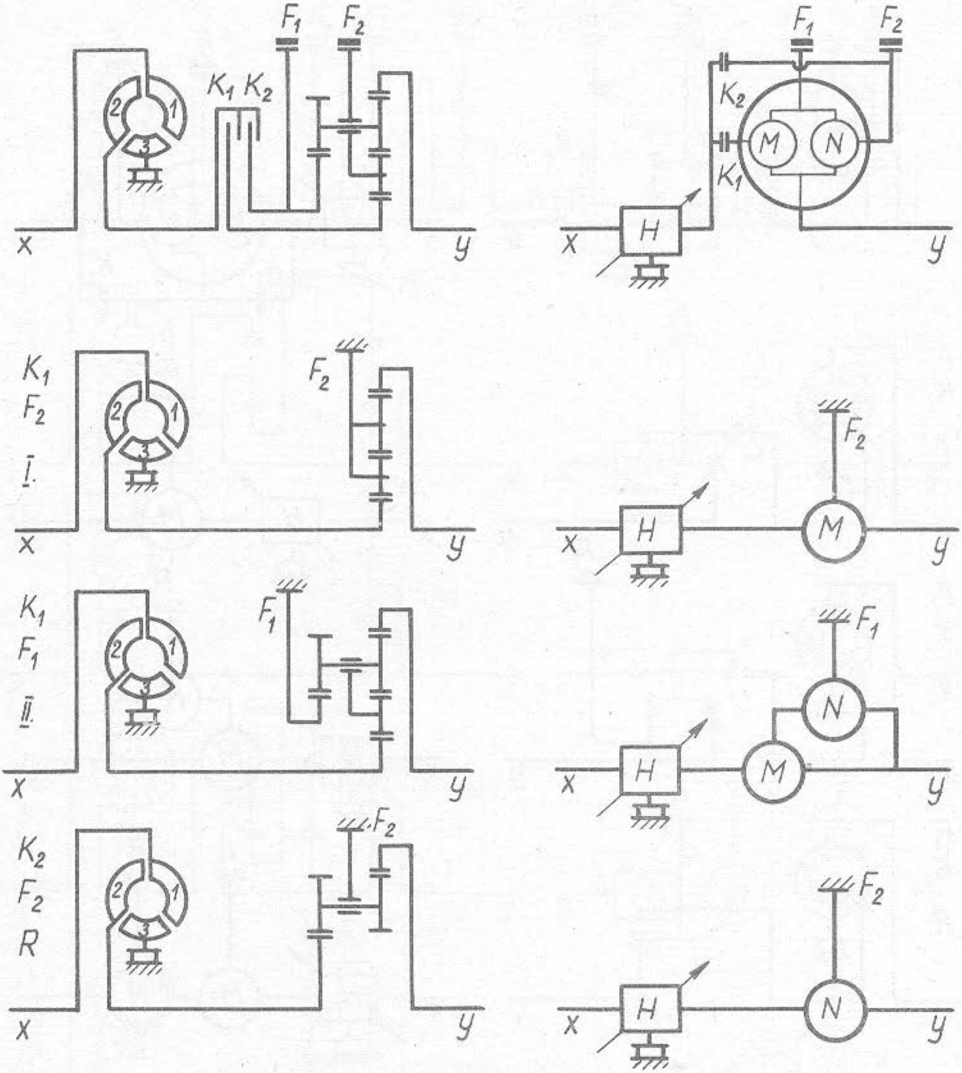


Fig. 161.

If the first member is shortclosed, the second member gives three stages. If the first member is fastened, together with the second member again three stages are obtained, finally the first member alone (the second member shortclosed) gives the seventh one. In Fig. 162 (because of the lack of space) only five stages are indicated, the lacking two can be obtained (both are reversed ones) by the drawing in of the F_4 with the simultaneous coupling in of K_1 resp. F_1.

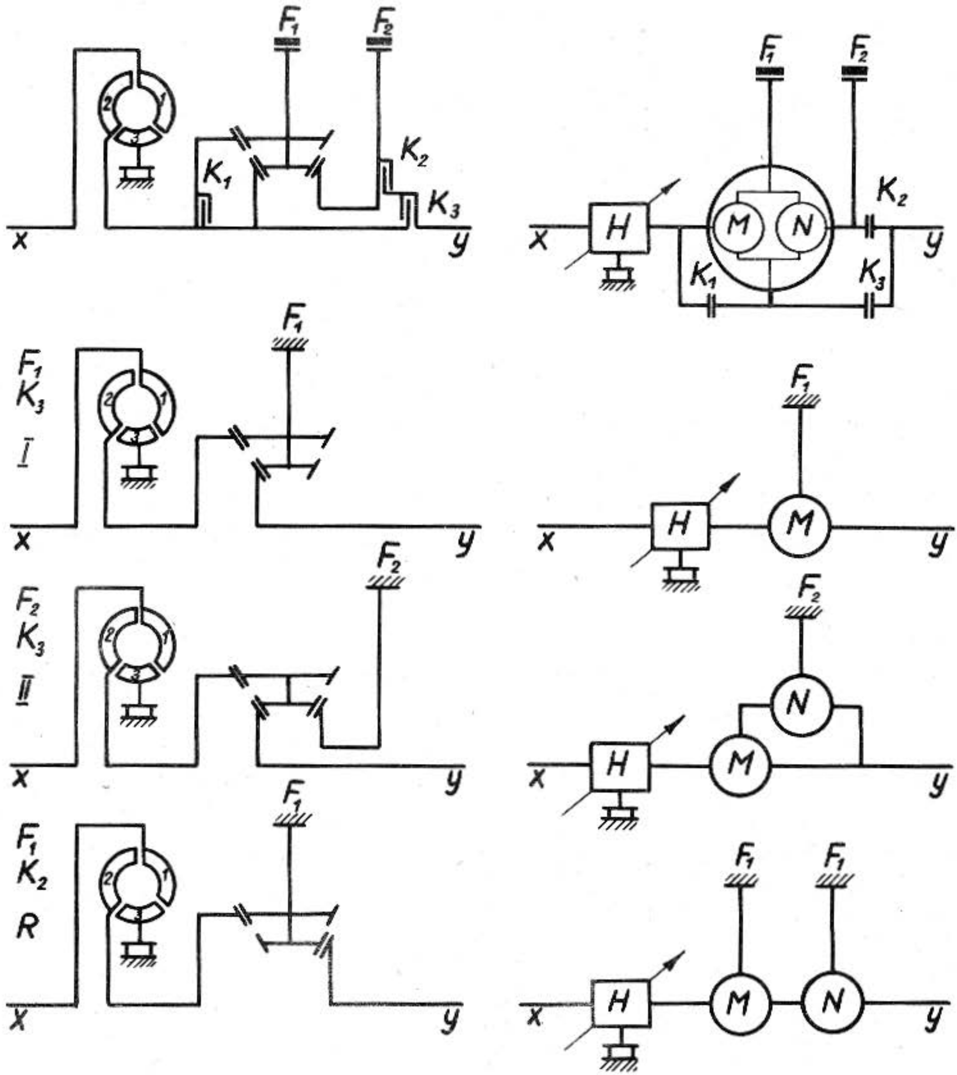


Fig. 162.

In the previous chapter was mentioned that there are hydraulic elements having two (or more) outgoing shafts. These, of course, must be united, but before this a simply fastened epicyclic gear can be inserted (Fig. 145).

In Fig. 164 for the first moment a such case seems to be shown, in the reality however, the case is that of the three running wheels being in the torque converter two, the guide wheel and the turbine wheel change their roles at the change-over from the one stage to the other. Essentially, both stages are simple series connected hydraulic and mechanical elements [71].

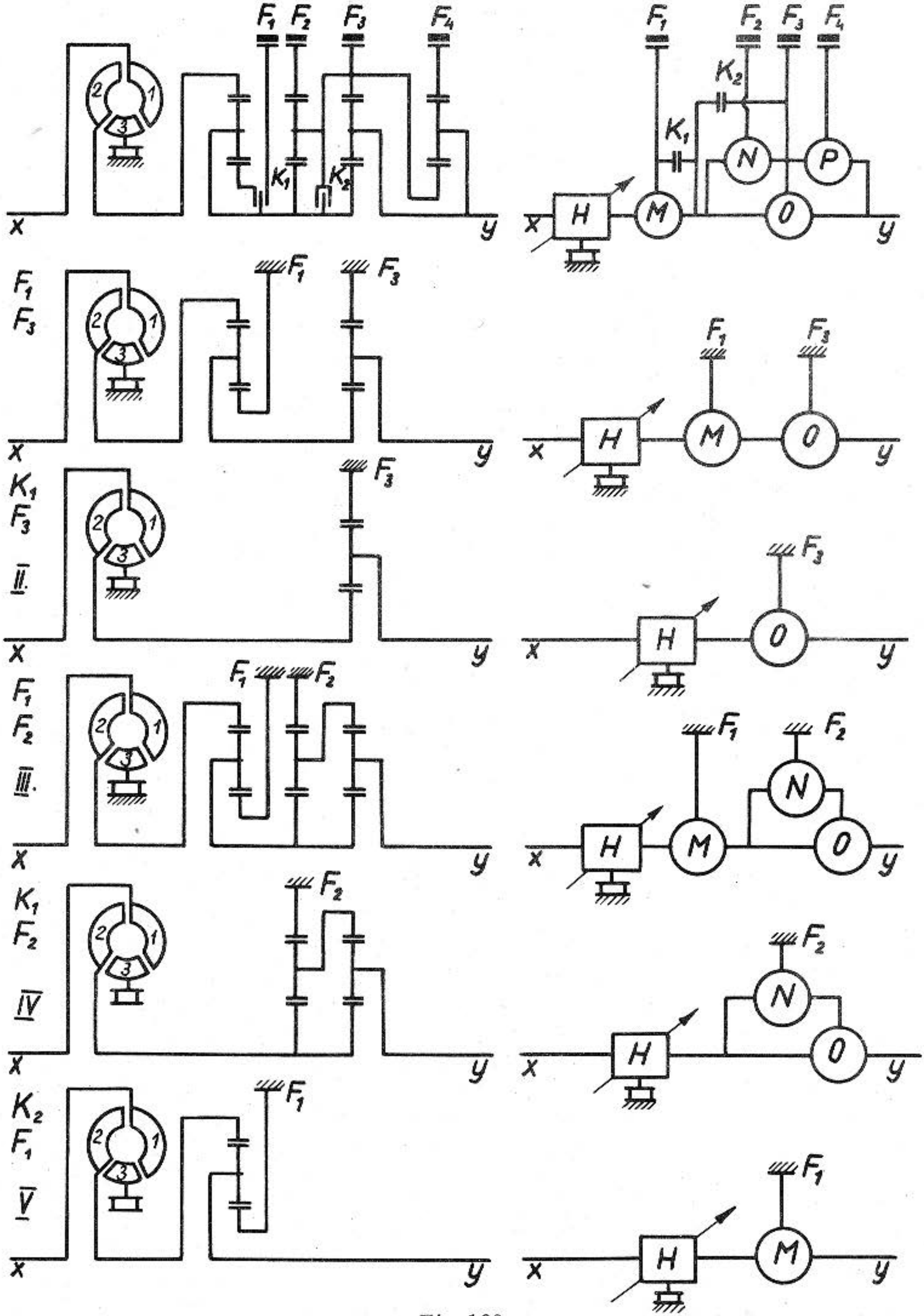

F1 F2 F3 F4
K1 K2
1 2 3
N M O P H
x y
F1 F3
K1 F3 II.
F1 F2 III.
K1 F2 IV
K2 F1 V

Fig. 163

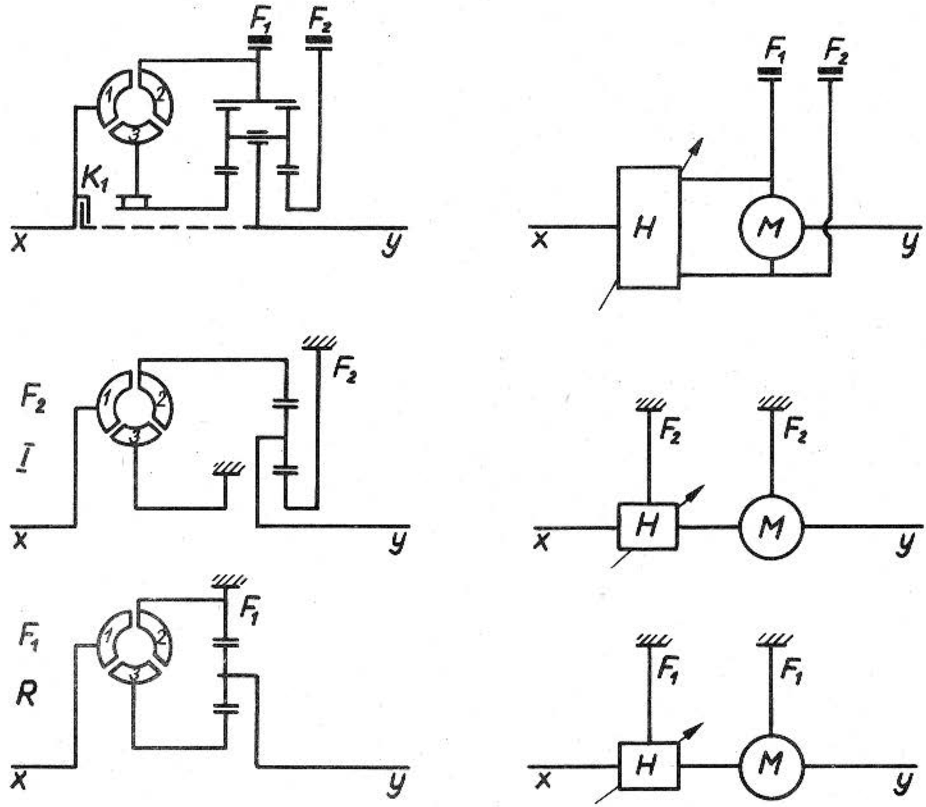
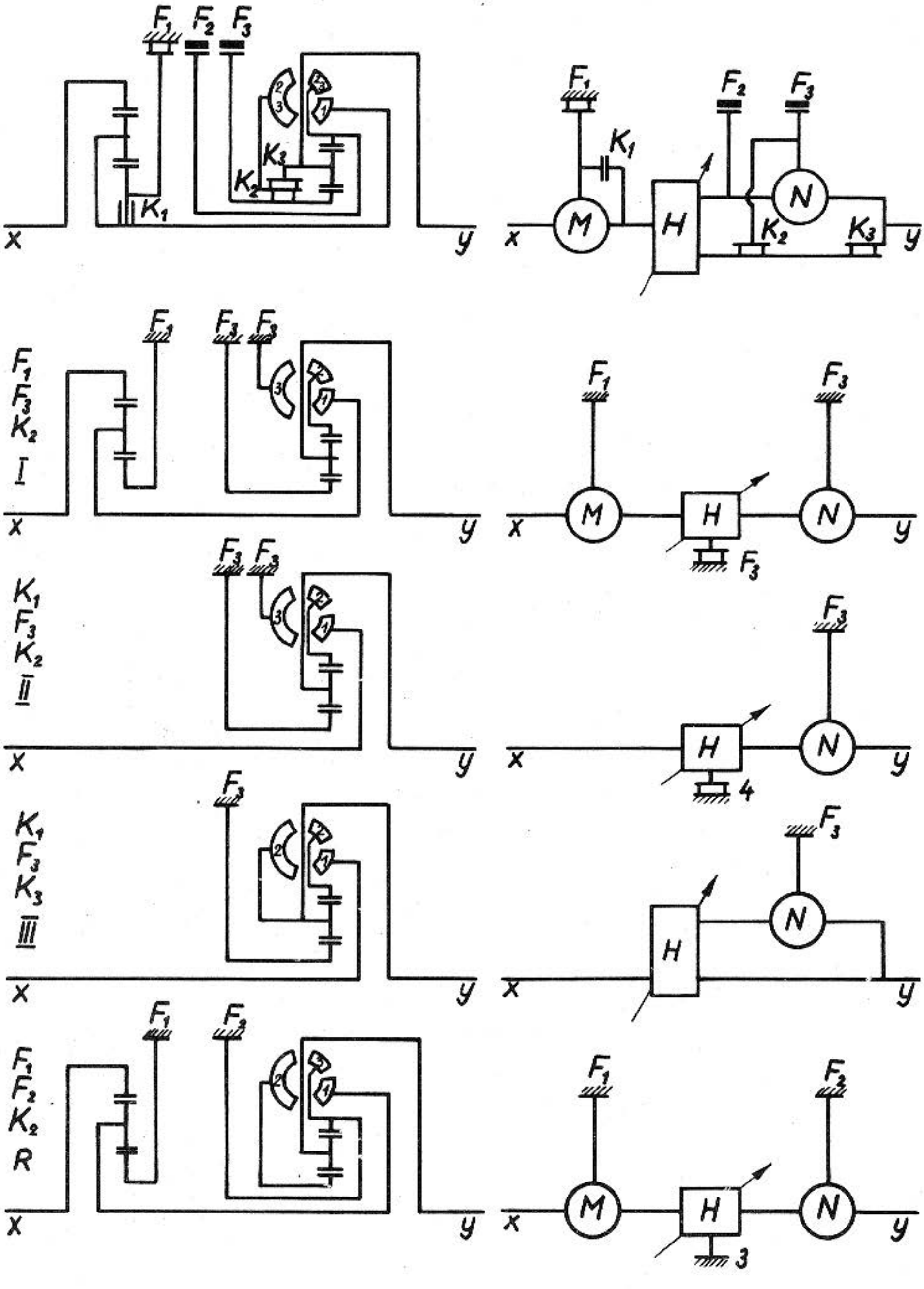


Fig. 164.

In Fig. 165 the hydraulic element really has two outgoing shafts [80]. In the first stage two simply fastened epicyclic gear flank one hydrodinamical torque converter (the other outgoing shaft of the hydraulic element is fastened here), the fourth stage is of the same design, only here the other shaft of the hydraulic element is fastened and as a consequence the binding-in of the epicyclic gear N (i.e. its transmission) was also changed. In the second stage only one simply fastened epicyclic gear is series connected to the hydraulic element. Only in the third stage can be the case met with that the shafts coming out of the hydraulic element are united, before this, however, into one of the shafts a simply fastened epicyclic gear is inserted.

Of the two stages of Fig. 166 the second one is similar to that shown in the preceding Fig., the first, tough, differs from it inasmuch, that before uniting the shafts coming out of the hydraulic element, not a simply fastened epicyclic gear but one prefastened by a simply fastened epicyclic gear is inserted into one of the shafts [90].

The driving gear seen in Fig. 167 has two hydraulic elements with outgoing shafts in all their four stages, and to this are four complex epicyclic gears



F1 F2 F3
K3
K2
K1
x
y
F1
K1
F2
F3
M
H
N
K2
K3
x
y
F1
F3
K2
I
F1
F3 F3
x
y
F1
F3
M
H
N
F3
x
y
K1
F3
K2
II
F3 F3
x
y
F3
H
N
4
x
y
K1
F3
K3
III
F3
x
y
F3
H
N
x
y
F1
F2
K2
R
F1
F2
x
y
F1
F2
M
H
N
3
x
y

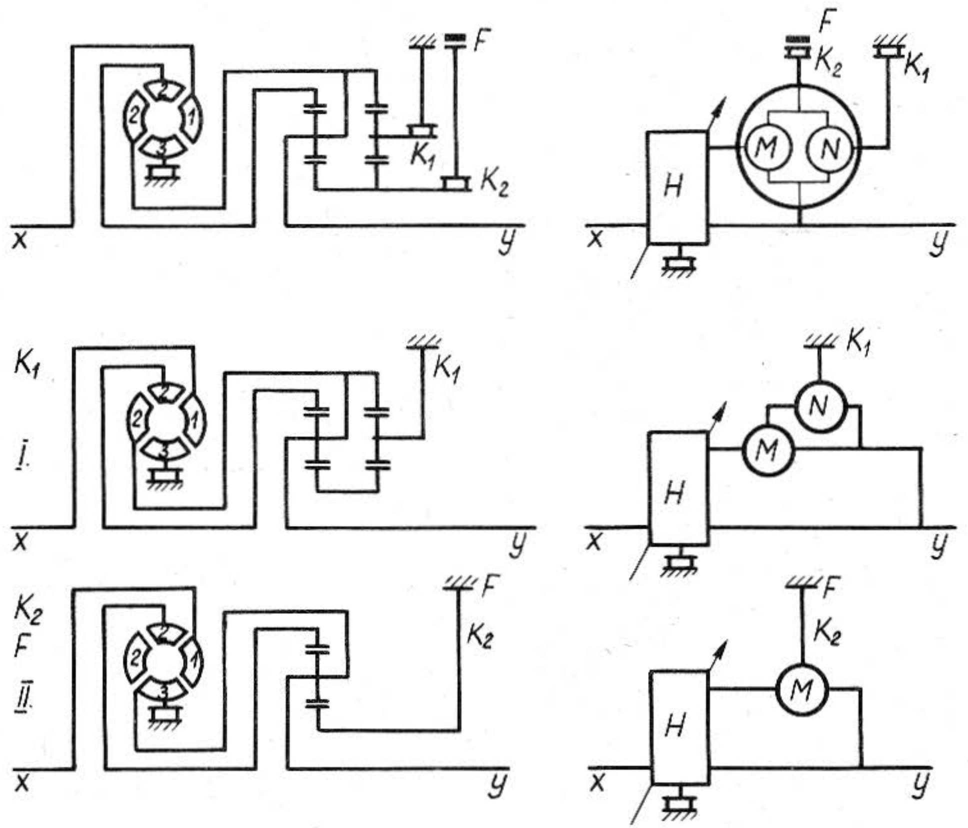
Fig. 165

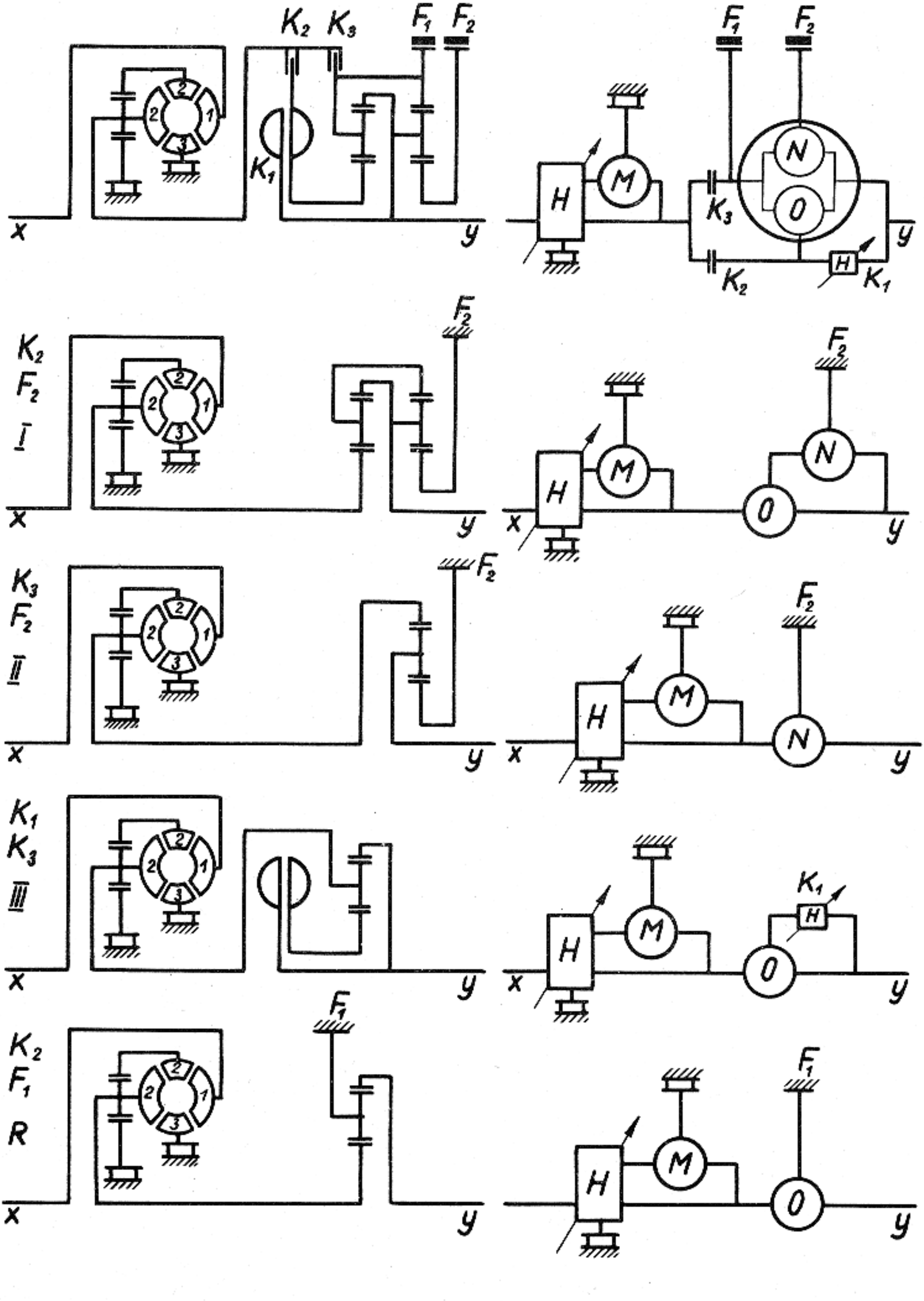
of different transmission series connected: in the first stage an epicyclic gear prefastened by a simply fastened one, in the second stage a simply fastened epicyclic gear, in the third stage an epicyclic gear prefastened by a hydromechanical clutch and in the fourth stage a simply fastened epicyclic gear again.

In Fig. 168 a doubly complex (moreover, a concjunct) epicyclic gear is seen (see also Fig. 68) of which a simply fastened simple epicyclic gear will become in stages 1, 2 and 5, resp. a simple epicyclic gear backfastened by a hydraulic element in stages 3 and 4. Between the 1 and 2, resp. the 3 and 4 stages the difference is rendered by the epicyclic gear M fastened before the hydraulic element.

In Fig. 169 essentially the same change-speed gear is seen, only the design of the concrete epicyclic gear is else, further the epicyclic gear M fastened before them is lacking [70].

Again two similar, moreover, in their theoretical design completely identical change-speed gears are presented in Figs. 170 and 171. The one is a Renault [103] the other a Brokhause-Salerni [51] change-speed gear. Not much





K2 K3 F1 F2
K1
x y
F1 F2
M H N O
K3 K2 H K1
x y
K2
F2
I
F2
x y
F2
M H N O
x y
K3
F2
II
F2
x y
M H F2 N
x y
K1
K3
III
x y
M H K1 H O
x y
K2
F1
R
F1
x y
M H F1 O
y

Fig. 167

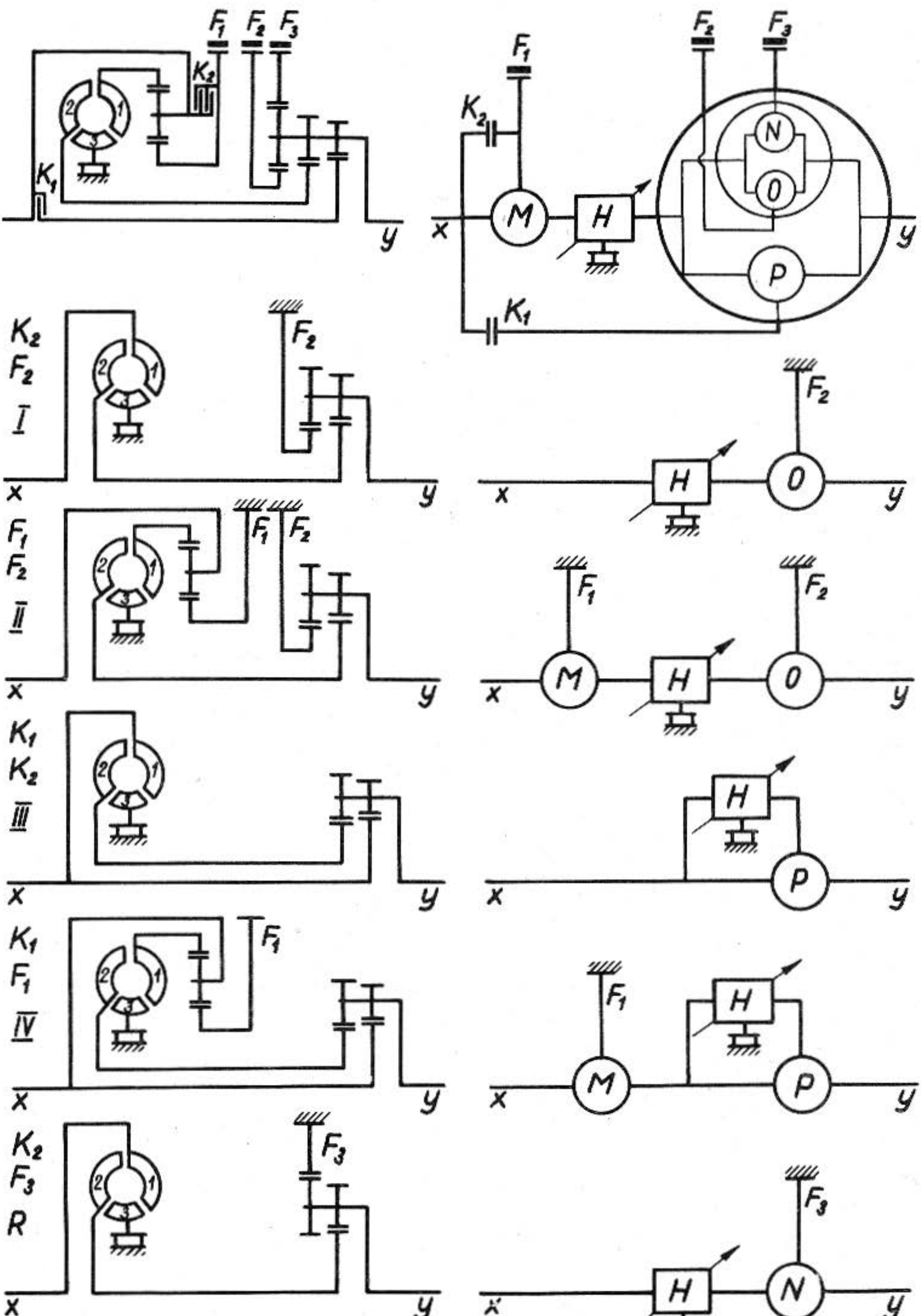


Fig. 168

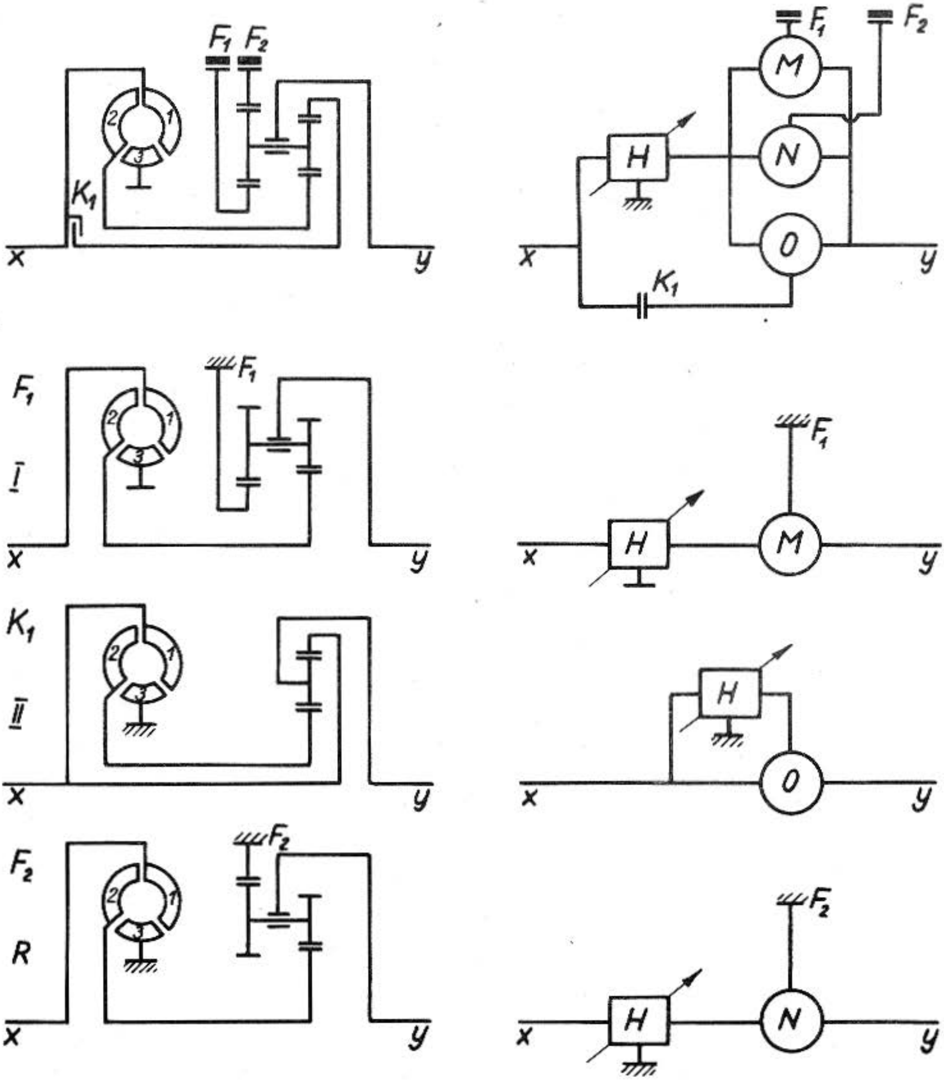


Fig. 169.

more complicated is the driving gear seen in Fig. 172 [72] the hydraulic element of which shows some affinity to the hydraulic element described in Fig. 163.

In Fig. 173 the theoretical design of the well kown Hydromatic [103] change-speed gear can be seen, while in Fig. 174 the more recent type of the same is presented. The Figs. need no more ample explanation.

It is again a change-speed gear of two members, which under the name Diwabus [53 , 54] is shown in Fig. 175. Here the second member corresponds to the 2 basic type of Fig. 72.

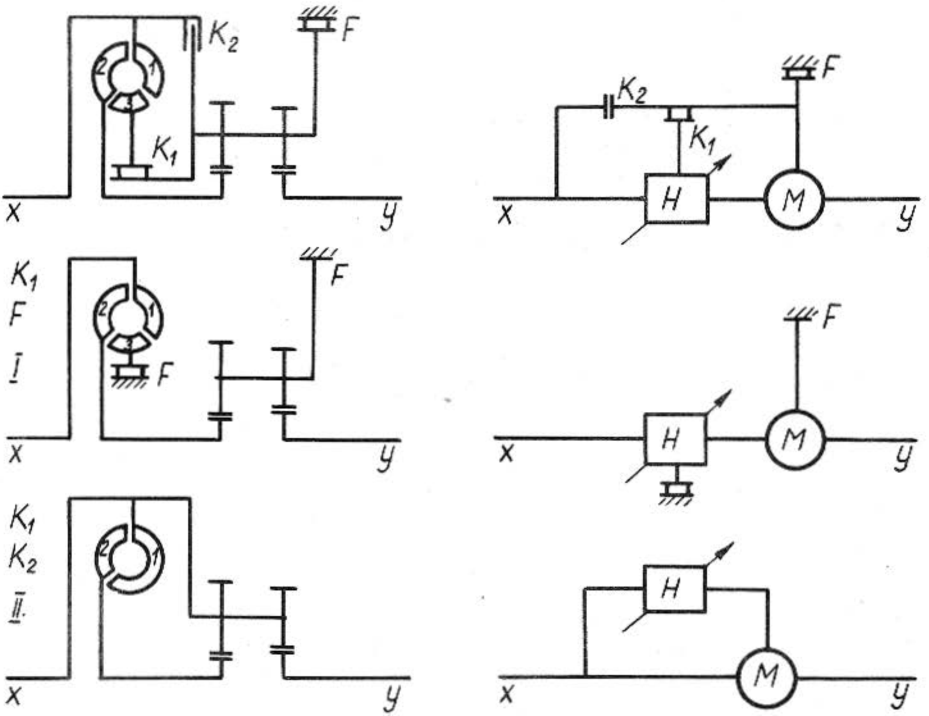


Fig. 170.

Finally, two change speed gears of special design will be described. At the one the hydraulic element has three outgoing shafts (Fig. 176) [81] and an epicyclic gear of two degrees of freedom connects, as an equalizing gear, the middle shaft to the two extreme ones. Otherwise, the latter are, in the customary manner, after an inserted simply fastened epicyclic gear, united with each other. The other change-speed gear's (Fig. 177) [93] specialities are the second, third and fourth stages. In the second stage the unification of the two outgoing shafts of the hydraulic element is performed in the epicyclic gear (N) of two degrees of freedom, used as equalizing gear. In stage third there are two epicyclic gears backfastened by the hydraulic element, the hydraulic connecting elements of the two epicyclic gears, however, are united again, resp. are provided with a common ingoing shaft.

This may be said also in other words so, that both series connected epicyclic gears were backfastened by a single hydraulic element having two outgoing shafts.

In the fourth stage two epicyclic gears of two degrees of freedom (the N and M) figure as an equalizing gear, which are so to say "shunting" the two outgoing shafts of the hydraulic element. Just because of the fact that they are only "shunting" it, separate care must be taken for the final connecting of the shafts. This occurs after the simply fastened epicyclic gear O.

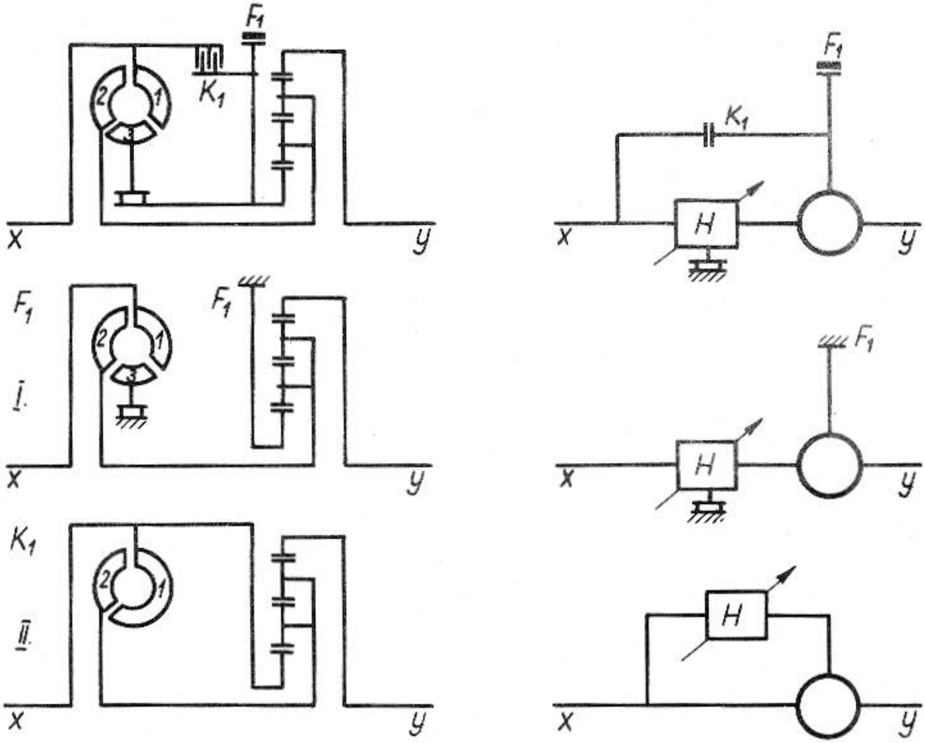


Fig. 171.

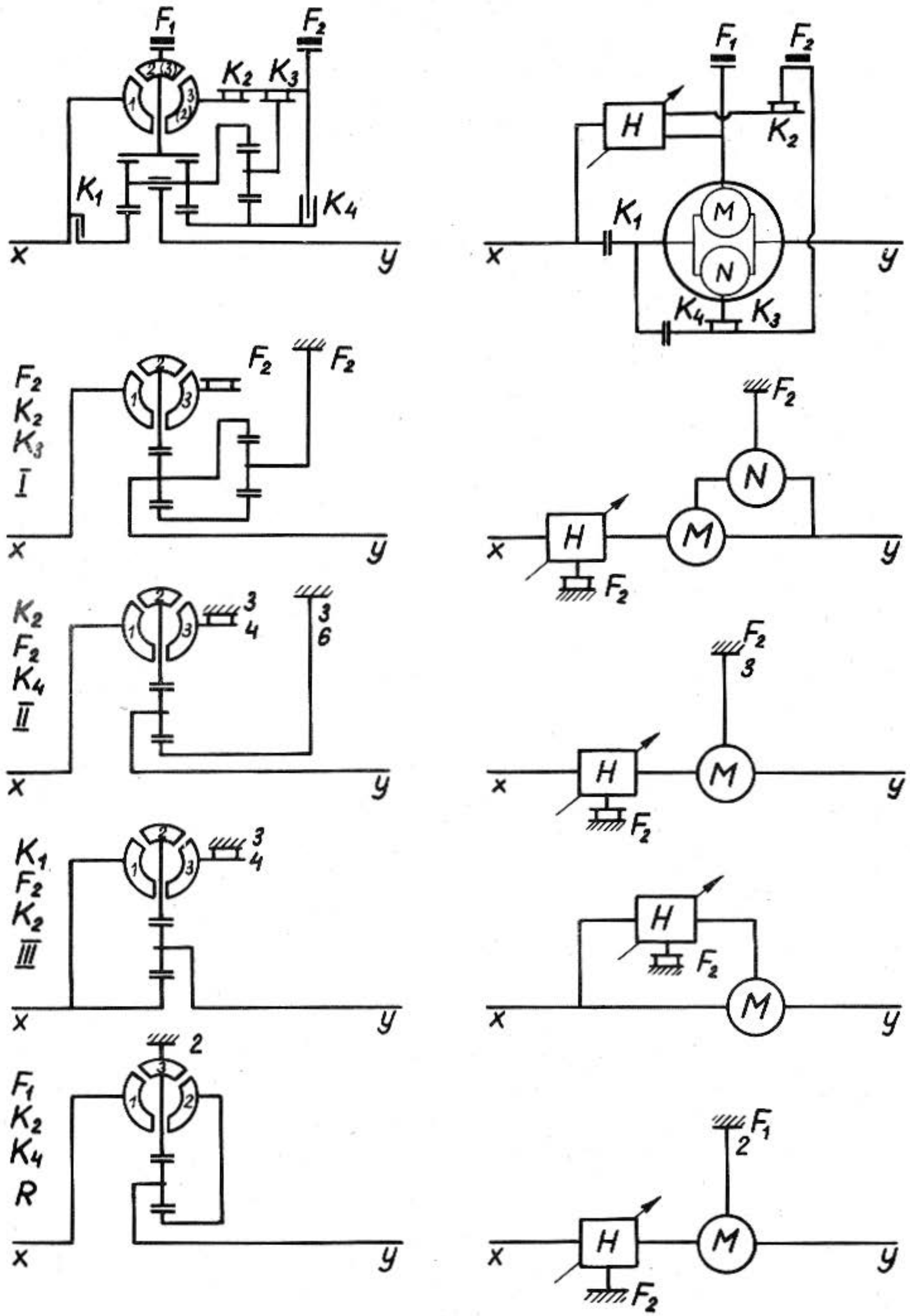

F1
F2
K2
K3
K1
K4
1
2 (3)
3 (2)
x
y
H
M
N
F2
K2
K3
I
K2
F2
K4
II
3
4
6
K1
F2
K2
III
F1
K2
K4
R
2

Fig. 172.

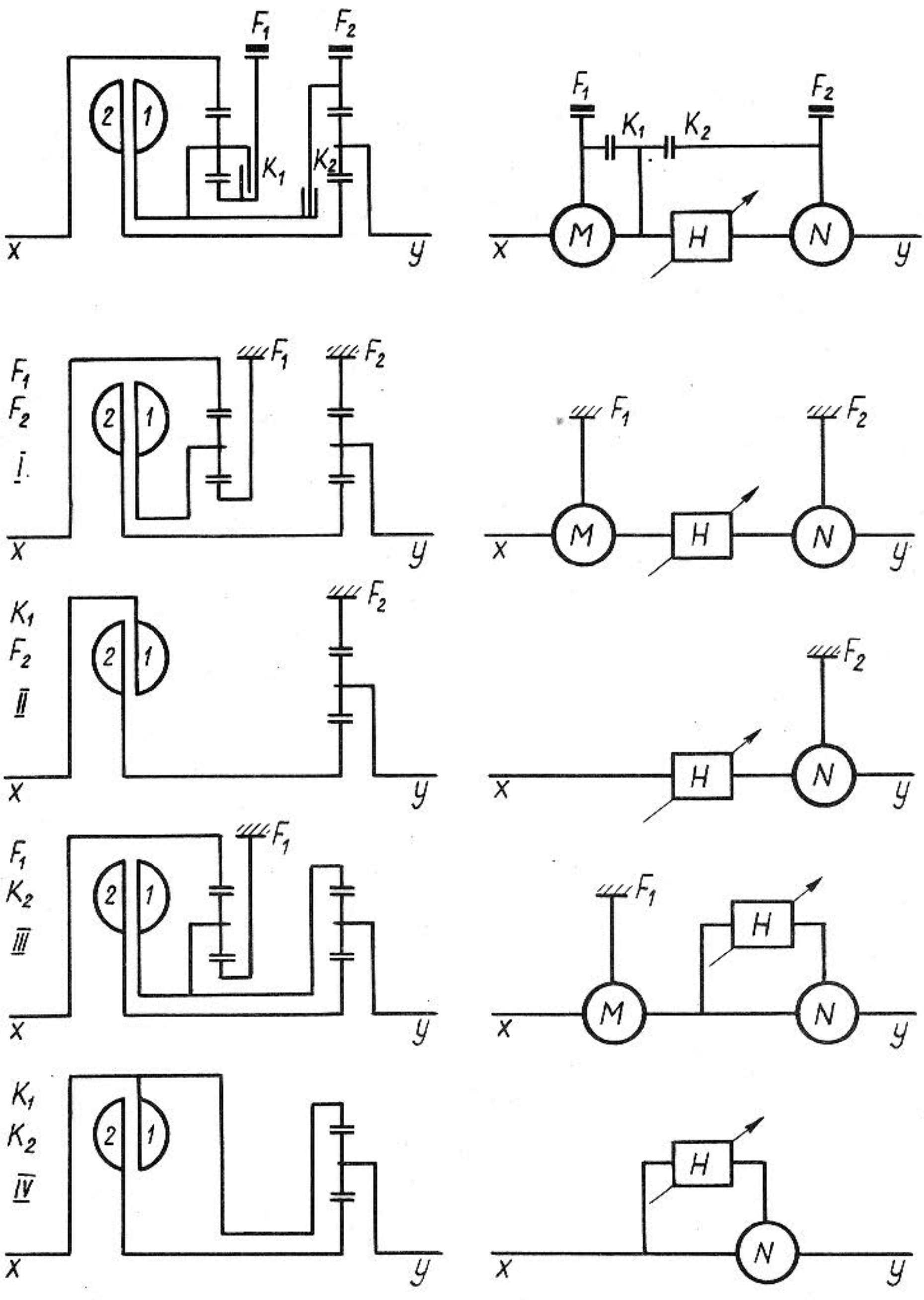


Fig. 173.

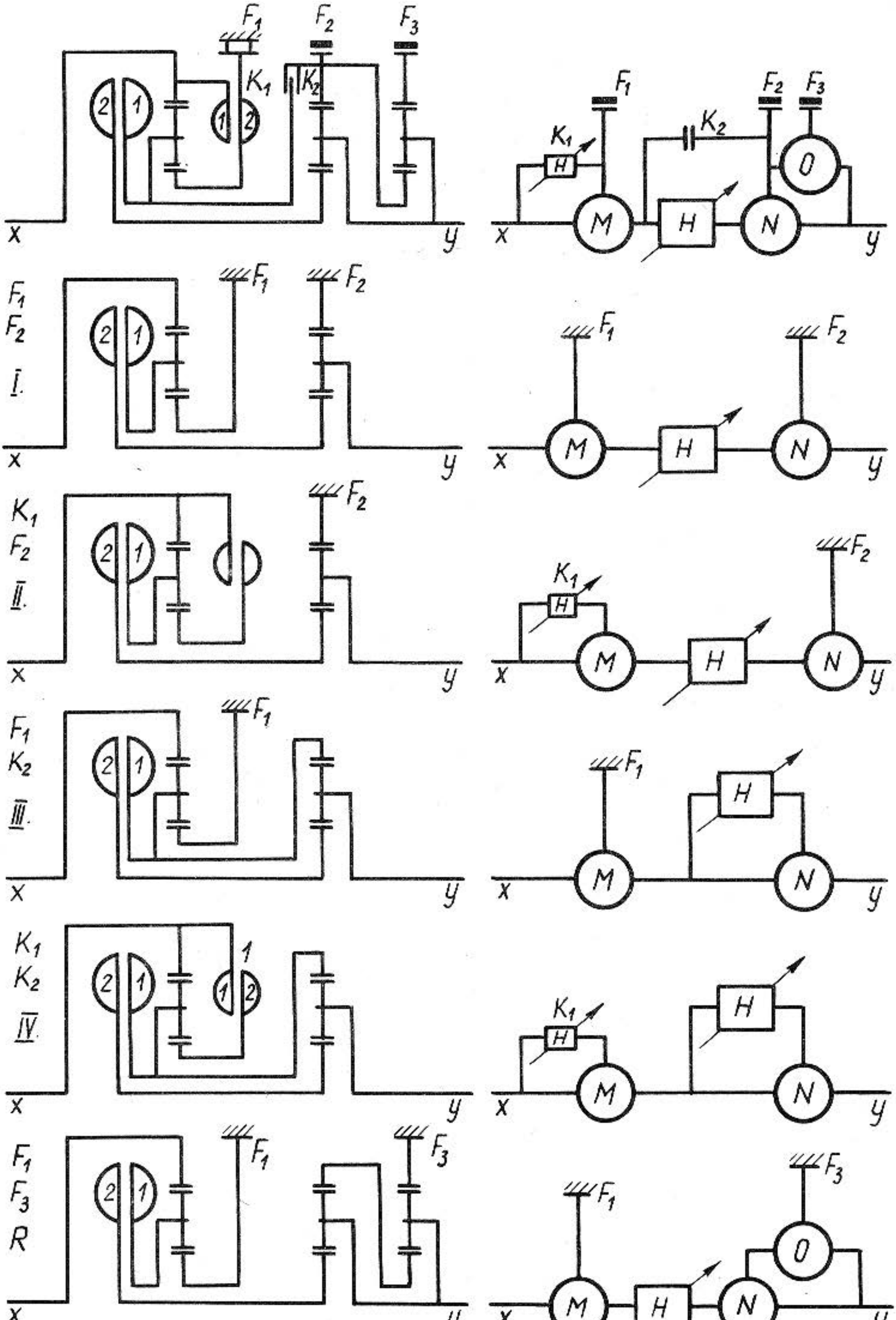


Fig. 174.

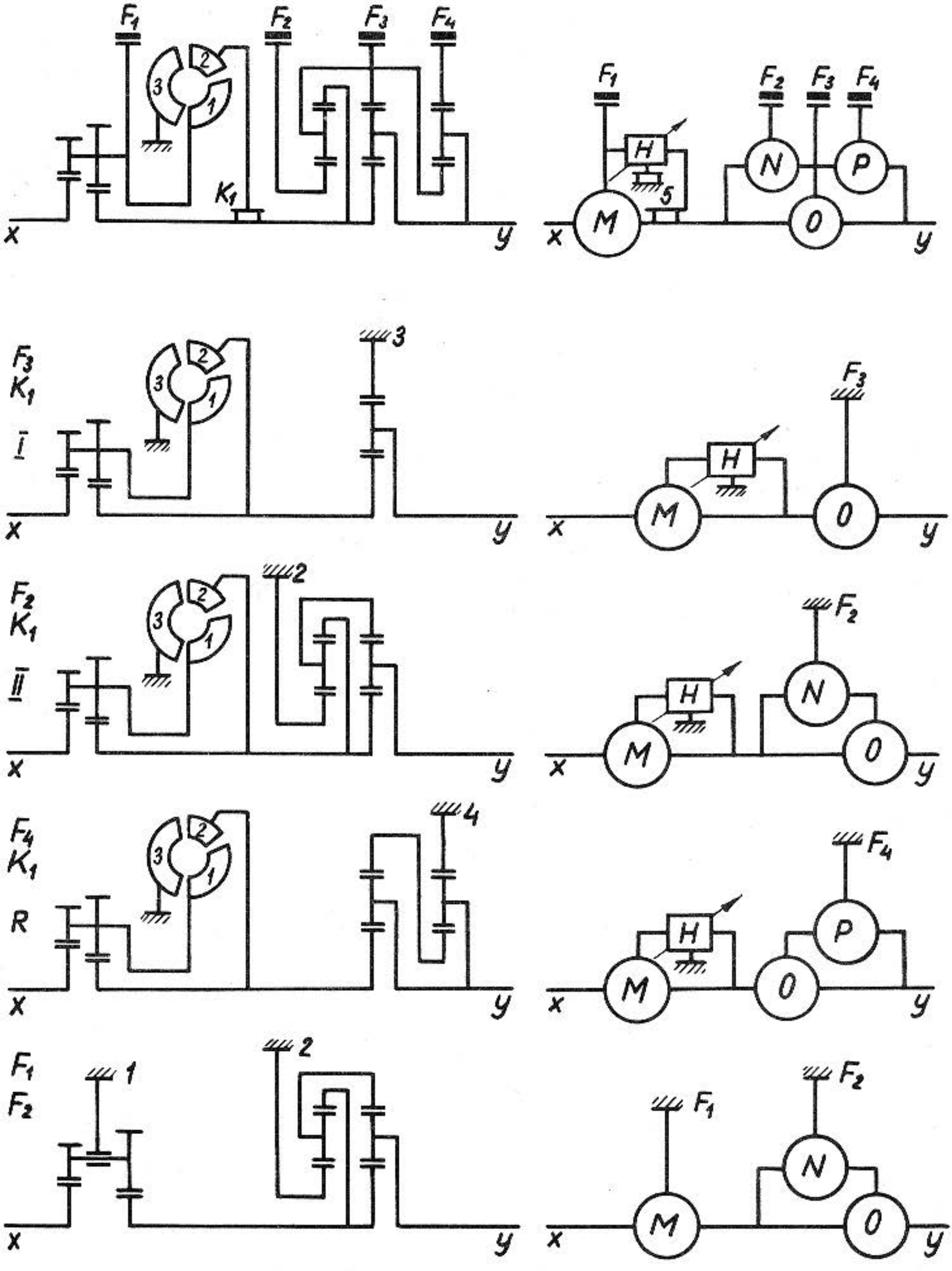


Fig. 175.

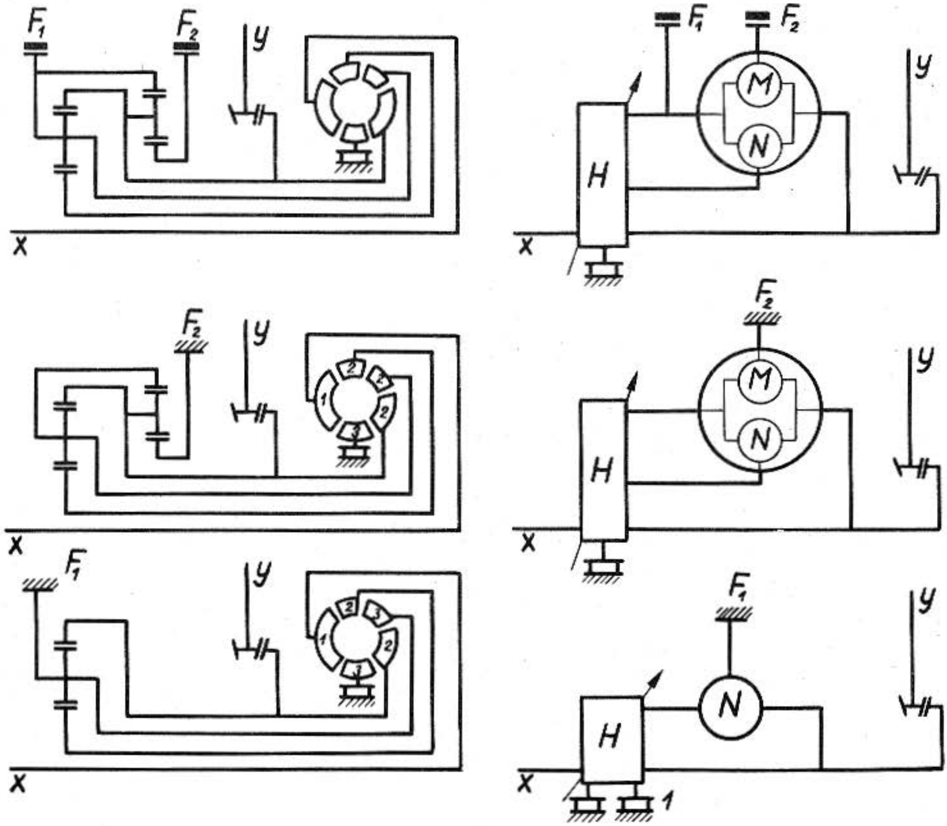


Fig. 176.

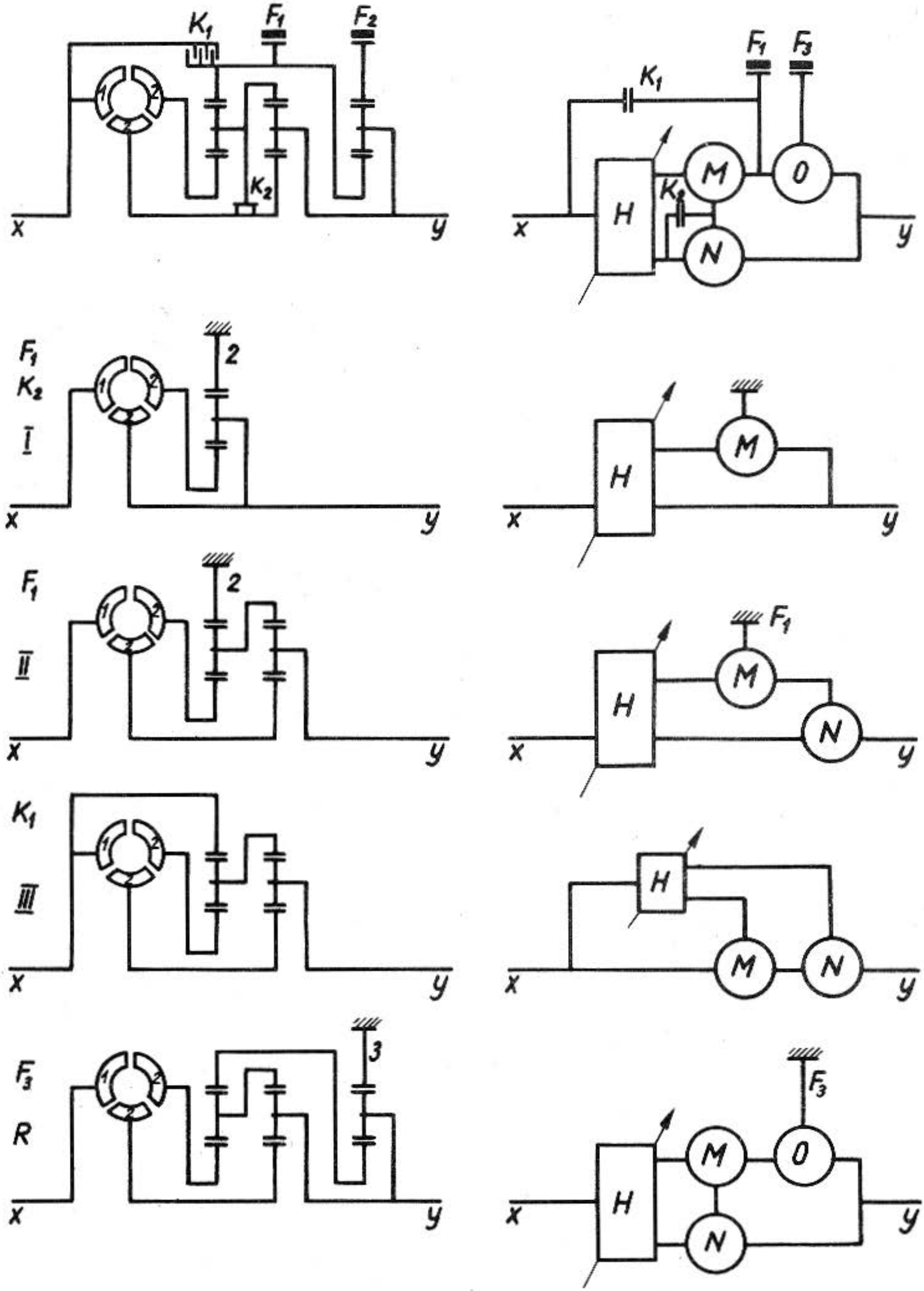

K1
F1
F2
K2
x
y
F1
F3
K1
M
O
H
K2
N
F1
K2
I
2
M
H
F1
II
2
M
N
K1
III
H
M
N
F3
R
3
F3
M
O
H
N

Fig. 177.

same P.G.T. types are shown as in sketches 9 back to 1, but with reversed 1 and 2 indexes.

On the basis of Fig. 15, it can be stated that two types of elementary P.G.T. and four types of simple P.G.T. can be derived from type 14b, each with two varieties of connection. If one made the same process of derivation for types 14a and 14c, one would have gotten the same types of P. G.T. but with two other couples of varieties of connection.

The types of elementary and simple P.G.T. derived from the basic type, shown in Fig. 10, are illustrated in Figs 16 and 17. These types are all without auxiliary planet gears. If one starts from the basic type given in the top of Fig. 18, instead of Fig. 10, one will get all types of the elementary and simple P.G.T. *with* or *without* auxiliary planet gears. The process is similar to the process illustrated above, so only the main conclusions will be reported.

The basic type given in Fig. 18 has four central gears, two of them have index y (y' and y''). At the same time, one can take into account only one of elements y; otherwise the mechanism becomes overspecified. The left side of Fig. 18 corresponds to the well-known sketches 14*a*, 14*b*, and 14*c*. The right side of Fig. 18 illustrates the basic type of P. G. T. with auxiliary planet gears in three varieties of connection.

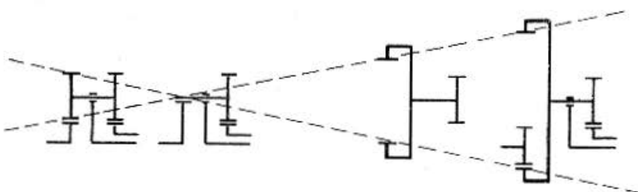


Fig. 20

If one takes into account element y' the formula, convenient to Formula 11, is as follows:

$$B = \frac{D_{4z} + \dfrac{D_z}{D_{y'}} D_{4y'} \dfrac{D_{4zx}}{D_{4xz}}}{D_{4z} + \dfrac{D_z}{D_x} D_{4x} \dfrac{D_{4zx}}{D_{4xz}}}. \tag{15}$$

If one takes into account element y'',

$$B = \frac{D_{4z} - \dfrac{D_z}{D_{y''}} D_{4y''}}{D_{4z} + \dfrac{D_z}{D_x} D_{4x} \dfrac{D_{4zx}}{D_{4xz}}}. \tag{16}$$

SUMMARY

The literature of the epicyclic gear and of the change-speed gear with epicyclic gear deals in the first place with the theoretical and constructional questions of the simple epicyclic gear, in a less extensive degree with the theoretical relations of the complex epicyclic gears, further, with the description of the produced change-speed gears. We encounter also the theoretical examination of the change-speed gear stages; the theory of the change-speed gear itself, however, is not elaborated at all up till now.

In our study we endeavoured to explore the up till now inelaborated parts of the theory of the epicyclic gear and mainly that of the change-speed gear with epicyclic gear, to fill up the gaps and to work out such a comprehensive theory which comprises in an organic system the epicyclic gears and change--speed gears with epicyclic gears, calls the attention to the organic relations between the individual concrete shapes of representation and secures the possibility for the exploring of the up till now unknown formes of presentation of the system.

We started from the examination of the elementary and simple epicyclic gear. That most general shape of the simple epicyclic gear was determined to which all existing concrete epicyclic gears can be led back (Figs. 8 and 9), be it of a front- or bevel gear design. For the kinematical examination of the simple epicyclic gear the analytical method of Willis and the graphic one of Kutzbach was selected. The latter, however, needed further development, since the original method was not adequate for the large family of the epicyclic gear with auxiliary epicyclic wheel. The proposed new method differs inasmuch from that of Kutzbach that the peripheral speed vector of the central wheel not connected directly to the main epicyclic wheel was not taken on its real periphery, but in the point intersected by the straight laid across the points of connection of the auxiliary epicyclic wheel (Fig. 16 or 17). In the course of the deduction some important statements were made regarding also the kinematical role of the auxiliary epicyclic wheel.

As a result of the dynamical examination of the simple epicyclic gear, proportions suitable for good handling were obtained for the torque- and performance conditions (29), (30). In course of the examination it was stated on the one hand, that the kinematical basic transmission characterizes unambigouosly also the dynamical conditions and on the other, that, as opposed to the two kinematical degrees of freedom, the dynamical conditions are already made determined by taking a simple torque resp. performance.

In the end of the chapter the definitions of the complex and conjunct epicyclic gears were presented.

Before dealing with the complex epicyclic gear, by which a simple or complex epicyclic gear completed by some connecting element is understood (the connecting element can be any kind of power transmission structure) at first the generalization of the simple and complex epicyclic gears was performed which, after all, meant the further development of the Willis method. Instead of the kinematical basic transmission used up till now (4) the general kinematical basic transmission (31) was introduced which characterizes unambigously the epicyclic gear, independently of type, geometrical, dimensional proportions and manner of binding-in, both from kinematical and dynamical viewpoint (32), (33), (34). The generalization was performed also for the complex epicyclic gear, as a result of which two general kinematical basic transmissions were obtained (38), (40) which may be handled in the same manner as the kinematical basic transmission of the simple epicyclic gear (41-44).

Since between the incoming and outgoing shafts of the change speed gears unambigous kinematical and dynamical conditions have to prevail, the epicyclic gear originally of two degrees of freedom must be transformed to one of one degree of freedom. There are two ways for fastening one of the degrees of freedom of the simple epicyclic gear: one of its shafts will be connected either to the base or to one of its other two shafts (in the former case we deal with the simply fastened epicyclic gear, Fig. 28, in the latter with the transfastened one, Fig. 29). In the latter case we can speak of the prefastened or of the backfastened epicyclic gear, according to the direction of the drive. At complex epicyclic gears outside of the simply fastening and transfastening there is a possibility of crossfastening, too (Figs. 35 and 36). For the simply fastening, transfastening and crossfastening a connecting element is used, that is why they are called complex epicyclic gears. Apart from this, in any of the shafts may a connecting element be inserted (31-32., 37-41 Figs.), moreover, if necessary, also even more.

Examining the kinematical conditions of the complex epicyclic gear, the formulae of the kinematical transmissions of all types were determined, for the general cases as well as for the simple ones (55), (59), (66), (69), (84).

At the examination of the dynamical conditions at first the formulae of torque-transmission were determined which can also be obtained directly from the corresponding kinematical formulae by the substitution of $i = -\frac{1}{k}$, resp.

$$\frac{\omega_\infty}{\omega_0} = -\frac{M_0}{M_\infty}$$

After this the performance-flow was dealt with, as a result of which such a simple formula was obtained (117) by the calculation of which the performance-flow conditions of the transfastened epicyclic gear presently became known, be it a case of an epicyclic gear or connecting element of whatever type. Simple expressions were obtained also for the examination of the performance-flow of the crossfastened epicyclic gear (143), (144), (145), moreover, by the help of the expressions told, the extent and direction of the performances flowing within the complex epicyclic gear can be easily determined.

As a result of the analytical examination of the epicyclic gear, therefore, such expressions resp. formulae were obtained, by the help of which the kinematical, torque- and performance conditions of any epicyclic gear structure can be examined simply and rapidly. After this the elaboration of the theory of the change speed gears with epicyclic gears was commenced.

The complex epicyclic gears were examined also from the viewpoint of their suitability of applying them in the change speed gear as its stages. Such complex epicyclic gears in which all connecting elements are mechanical power transmission sturctures, may be change-speed gear stages.

The most simple mechanical stage is the epicyclic gear fastened by the frictional clutch. (Fig. 49). If this same connecnting element is used for transfastening, then we obtain the "direct" stage (Fig. 50), since the epicyclic gear will be shortclosed (blocked).

Since the epicyclic gear fastened by closed brake will be transfastened to a mechanical power transmission structure of constant transmission and of one degree of freedom, it can be used as a connecting element of further epicyclic gears. So e.g. the simply fastened epicyclic gear can also be the connecting element of a transfastened epicyclic gear (Fig. 50). But also the transfastened epicyclic gear is transfastened to a mechanical structure of constant transmission and of one degree of freedom, if a mechanical connecting element (e.g. a simply fastened epicyclic gear) is applied, therefore also this can be used as the connecting element of further epicyclic gears (Fig. 57). The same can be told also about the complex epicyclic gears. After all, in consequence of the property of the epicyclic gears that they can be accumulated, a change-speed stage of a given transmission can be realized by an infinite number of epicyclic gear combinations.

All conmibations, however, can be led back to the three basic types described in the foregoing.

The different combinations of the complex epicyclic gears used as stages constitute the change-speed gear.

If all three members of the epicyclic gears are in constant connection to any three of the main elements of the change-speed gear (the incoming and outgoing shafts as well as the brake drums), then we speak of a change-speed gear of constant inside design the main property of which is that the needed and sufficient number of the simple epicyclic gears is equal to the number of the stages differing from the 1:1 and the same number of brake drums is also needed. According to these the base of the change-speed gear of one stage is the simple epicyclic gear, that of the one of two stages the complex epicyclic gear, while of the one of three stages the doubly complex epicyclic gear (Figs. 67-69) etc. To connect the epicyclic gear members to the main elements of the change-speed gear in case of one stage there is only one kind of possibility (Fig. 63) but in case of one of two stages there are three kinds of possibilities (Fig. 66), while in case of one of three stages there is a possibility already of 14 variations (Fig. 72), etc. etc. For studying the variational possibilities of the multi-stage change-speed gears a simple method was elaborated (Figs. 73-78), moreover, the theoretical scheme of the multi-stage change speed gears was also given (Fig. 79).

If between the applied epicyclic gears and the main elements of the change-speed gear there is no constant connection, but between the latter and the epicyclic gear member permutation is possible, then we can speak of the change-

-speed gear of varying inside design the main property of which is that the needed and sufficient number of the simple epicyclic gears is less as the number of the stages differing from the 1:1, but the number of the coupling elements is more. The advantage, therefore, of the new type change-speed gear proposed by us consists of the possibility of epicyclic gear saving. In case of permutation, namely, by a single simple epicyclic gear theoretically 6 different transmissions can be realized, while by a complex one 9-18 (Fig. 81-83).

Epicyclic gears can be saved also then, when two, theoretically independent change-speed gears are series connected. Such type may be called as a change-speed gear of two members (Figs. 84-86).

We obtain a hydromechanical stage from the complex epicyclic gear then, if at least one of its connecting elements is some kind of hydraulic power transmission structure.

At the simply fastened epicyclic gear it is expedient to insert the hydraulic element in the first place into the incoming or outgoing shaft. The stage obtained in this way is practically the series connection of a mechanical and a hydraulic power transmission structure kinematical- and torque transmission of which can be simply determined (149), (150). The factor of torque take-up depends on that, wether the hydraulic element is inserted into the incoming or the outgoing shaft. In the first case the torque take-up is identical to the one of the hydraulic element itself (152), in the second case, however, it depends on the basic transmission of the epicyclic gear (156). Between the two cases, therefore the difference is such as if the dimension of the hydraulic element would be uniform.

To proceed with the examinations, otherwise, a sample-characteristic was taken for the hydraulic element and so the research could be extended also to such field which remained outside of the scope of the examinations up till now.

The most characteristics case of the transfastened epicyclic gear is that, when the hydraulic element is used just for the transfastening. Taking in consideration the reversibility of the binding-in of the drive as well as of the hydraulic element, for the complex epicyclic gear transfastened by the hydraulic element four basic types are obtained (Fig. 96). Each type. however, can be used only in a determined general kinematical transmission interval, since the direction of the performance-flow can not be considered as independent from the direction of the binding-in of the hydraulic element.

The kinematical characteristics of the complex epicyclic gear transfastened by the hydraulic element (Figs. 97-100) well illustrate the dependence of the hydraulic element as well as of the transmissional operational range of the complex epicyclic gear (as a hydromechanical stage) on the general kinematical stage) on the general kinematical basic transmission.

The outside characteristics of the complex epicyclic gear transfastened by the hydraulic element and figuring as a hydromechanical stage was also - on the example of the outside characteristics of the hydraulic element - determined and, therefore, this can be handled in the same manner as the hydraulic element itself. A further advantage of this new method is that the characteristics of all hydraulic and hydromechanical driving gears can be compared, easily evaluated and the modification of the characteristics can be followed, resp. examined when the modification is in process.

One of the most important curves shows the change of the torque transmission B. On the characteristics may be seen that at certain types, at certain values B, the outside characteristics of the complex epicyclic gear transfastened by the hydraulic element are identical to the outside characteristics of the hydraulic element itself, proving their organic connection.

The hydraulic element, standing alone, after all, as a change-speed gear stage, can be considered as a special case of the complex epicyclic gear transfastened by the hydraulic element.

The efficiency curves (Figs. 105-108) can be derived from the torque transmission characteristics.

An other important curve of the outside characteristics shows the varying of the curve of torque take-up (Figs. 109-112). It can be generally stated that the torque take-up of the complex epicyclic gear transfastened by the hydraulic element increases when a value resulting in better efficiency is selected. This also means that in this case a hydraulic element of smaller dimensions is needed.

The complex epicyclic gear transfastened by the hydraulic element was examined also for the case of applying more connecting elements, when beside of a hydraulic element one or more mechanical connecting elements of constant transmission were inserted into one of the shafts.

It was stated that the effect of the mechanical connecting element of constant transmission inserted into the outgoing or incoming shaft of the transfastened epicyclic gear was identical to the one exerted also upon the simply fastened epicyclic gear. After this the case of the mechanical connecting element series connected to the hydraulic element was examined and it was stated that by varying the transmission of he mechanical connecting element, the extent and the place of the maximal efficiency could be varied optionally at all four types (Figs. 118-121). The situation is especially advantageous at two types, since there the improvement of the efficiency is easily solved also towards the smaller transmissions.

Also the effect of that mechanical connecting element was examined, which was inserted into shaft II of the transfastened epicyclic gear. It was stated, that the effect of such a connecting element is identical to the one of two such connecting elements the one of which is series connected to a complex epicyclic gear, while the other to a hydraulic element, therefore it could substitute them (Fig. 127).

At examining the epicyclic gear crossfastened by the hydraulic element the fact became clear that the four types of the transfastened epicyclic gear are, after all, special cases of the crossfastened epicyclic gear, and reversed, the crossfastened epicyclic gear is the amalgamation of the four types, as components of the transfastened epicyclic gear. The importance with which the character of the crossfastened epicyclic gear depends on the general kinematical basic transmissions of the two simple epicyclic gears constituing the complex epicyclic gear. We may, in practice, by varying the value B, shape optionally the outside characteristics of the complex epicyclic gear crossfastened by the hydraulic element.

The abovesaid can be extended to the case of applying not hydraulic connecting elements, too, and this will give both for the transfastened and crossfastened epicyclic gears a new interpretation in general. Accordingly, all transfastened epicyclic gears are the special cases of the epicyclic gear crossfastened by some simply fastened one, or reversed, the general form of the simply fastened and transfastened epicyclic gears is the crossfastened one.

We can speak of a hydromechanical change-speed gear then, when among its stages at least one hydromechanical stage can be found. The theoretical design of the multistage hydromechanical change-speed gear depends in the first place on the number of the hydromechanical stages applied. Since here hydromechanical and mechanical stages may be applied together, there are more possibilities of variation for the theoretical design than there were at the purely mechanical change speed gears. Otherwise, for the examination of the theoretical design of the hydromechanical change-speed gear also the critical examination of the hydromechanical stages has to be performed, this, however, is already beyond the aims of the present study.

To conclude, some examples were presented for the change-speed gears with epicyclic gears.

TERMINOLOGY APPLIED

In our study a number of new concepts appears; for these new terminologies had to be procured. It seemed to be expedient to list separately the terminologies applied. For the concepts already known it was our intention to use the expressions used also up till now in the home literature and in the first place the ones defined by Terplán and Vörös. The new terminologies are marked by underlining. At composing the list, the sequence of occurence was taken for base. For the individual expressions also explanations are provided:

elementary epicyclic gear - consists of a central wheel and of the epicyclic wheel rolling down on it (epicyclic gears), the latter(s) is (are) supported on the crank-arm in bearings;

central wheel - a wheel having a common shaft with that of the epicyclic gear;

epicyclic wheel - a wheel with a shaft not coinciding with that of the central wheel, the points of which rolling down on the central wheel run along an epicycloid;

crank-arm - an epicyclic gear-part uniaxial with the central wheel and bearing the shaft of the epicyclic wheel;

simple epicyclic gear - am elementary epicyclic gear completed by a central wheel;

epicyclic gear member - the two central wheels and the crank arm;

<u>parallel epicyclic wheels</u> - epicyclic wheels connected directly to the same central wheel;

<u>series connected epicyclic wheels</u> - epicyclic wheels connected directly to each other;

<u>main epicyclic wheel</u> - that of the series connected epicyclic wheels for which the speed-diagram was plotted;

<u>auxiliary epicyclic wheel</u> - that of the series connected epicyclic wheels, which is connected to the main epicyclic wheel;

sun-wheel - a central wheel of outside connection (gearing);

annular-wheel - a central wheel of inside connection (gearing);

simple epicyclic wheel - both connections occur on identical diameters;

double epicyclic wheel - the two connections occur on differing diameters;

kinematical basic transmission - the quotient of the relative angular speeds of the two central wheels related to the crank arm;

kinematical basic equation - the relation between the angular speeds of the epicyclic gear-members and the kinematical basic transmission;

torque proportions - the relation between the torque exerted on the epicyclic gear members and the kinematical basic transmission;

performance proportions - the relation between the performances led to the epicyclic gear members, the angular speeds of the latter and the kinematical basic transmission;

complex epicyclic gear - two simple epicyclic gears when two members of each of them is connected;

conjunct epicyclic gear - a complex epicyclic gear, when the connected members are unified, i.e. the same epicyclic gear parts pertain to the both simple epicyclic gears;

doubly complex epicyclic gear - to the two members of a complex epicyclic gear two members of a third one are connected;

epicyclic gear shaft - the mathematical shaft of the epicyclic gear members;

general kinematical basic transmission - the quotient of the relative angular speeds of two selected shafts of the epicyclic gear, related to the third shaft, independently of that: which of the shafts to which of the epicyclic gear members pertains;

binding-in of the epicyclic gear - the order of the mutual coordination of the epicyclic gear members and shafts;

general kinematical basic equation - the relation between the angular speeds of the epicyclic gear shafts and the general kinematical basic transmission;

torque proportions in a general case - the relation between the torque exerted on the epicyclic gear shafts and the general kinematical basic transmission;

performance proportions in a general case - the relation between the performance led onto the epicyclic gear shafts, the angular speeds of the shafts and the general kinematical basic transmission;

driving-in - introducing of the performance;

driving-out - leading out of the performance;

double driving-in - the performance is introduced simultaneously on two shafts of the epicyclic gear of two degrees of freedom;

simply fastened epicyclic gear - one of the shafts of the epicyclic gear is connected to the base being at standstill, the other two sahfts are led out;

transfastened epicyclic gear - one of the shafts of the epicyclic gear is connected to one of its two shafts led out;

straight drive transfastened epicyclic gear - the driving-in is performed on the led out shaft left alone;

prefastened epicyclic gear - a straight drive transfastened epicyclic gear;

reversed drive transfastened epicyclic gear - instead of driving-in, driving-out is performed on the led out shaft left alone:,

backfastened epicyclic gear - a reversed drive transfastened epicyclic gear;

connecting element - any type of power transmission structure used for simply fastening or transfastening the shaft of the epicyclic gear;

complex epicyclic gear - an epicyclic gear with connecting element or elements;

crossfastened epicyclic gear - two of the four shafts of the complex epicyclic gear are connected to each other by the help of some connecting element;

straight crossfastened epicyclic gear - the crossfastened shafts are in connecting to a simple epicyclic gear each;

inverted crossfastened epicyclic gear - the cross fastened shafts are in connection to both simple epicyclic gears;

crossfastened epicyclic gear of changed binding-in of the shafts - of the cross fastened shafts the one is connected only to one of the simple epicyclic gears, while the other to both of them;

kinematical transmission - the quotient of the angular speeds of the incoming and outgoing shafts;

torque transmission - the quotient of the torques exerted on the incoming and outgoing shafts;

outside elements of the change speed gear - the incoming and outgoing shafts, as well as the brake drums;

change speed gear of constant inside design - the epicyclic gears built-in are in constant connection to the outside elements of the change speed gear;

change speed gear of varying inside design - the built-in epicyclic gears can be connected according to different variations to the outside elements of the change speed gear by the help of coupling elements;

change speed gear of two members - two series connected change speed gears, when at both of them the transmission can be changed independently of each other, left out of attention wether the two change speed gears are structurally separated or not;

hydromechanical stage - such complex epicyclic gear transformed to one of one degree of freedom at least one connecting element of which is a hydraulic power transmission structure;

hydraulic element of straight binding-in - the turbine shaft connected to the led out shaft of the transfastened epicyclic gear;

hydraulic element of reversed binding-in - the pump shaft is connected to the led out shaft of the epicyclic gear;

kinematical characteristics - the relation between the hydraulic element and the kinematical transmission of the hydromechanical stage:

outside characteristics - the dependence of the torque transmission, the efficiency and the factor of torque take-up on the kinematical transmission;

hydromechanical change speed gear - such change speed gear which has at least one hydromechanical stage.

NOTATIONS USED

- central wheels;
- crank arm;
- epicyclic wheel;
- kinematical basic transmission;
- general kinematical basic transmission;
- angular speed;
- diameter;
- radius;
- peripheral speed vector;
- torque (moment);
- performance;
- simple epicyclic gears;
- shafts of the simple epicyclic gear;
- united shafts of the complex epicyclic gear;
- shaft of the doubly complex epicyclic gear;
- the general kinematical basic equation of the complex epicyclic gear for the I-II-III_1, resp. for the I-II-III_2 shafts;
- led out shafts of the complex epicyclic gear;
- incoming shaft;
- outgoing shaft;
- shafts of the connecting element;
- kinematical transmission;
- torque transmission;
- the kinematical transmission of the connecting element inserted into the shafts I, II, III of the epicyclic gear M;
- the kinematical transmission of the connecting element inserted into the shafts O resp. ∞ of the complex epicyclic gear;

- connecting element of constant transmission series connected to the hydraulic element;
- kinematical transmission between the led out shafts of the complex epicyclic gear;
- hydraulic element;
- kinematical transmission between the outgoing and incoming shafts of the hydraulic element;
- number of gearings (of teeth);
- number of stages;
- coupling element;
- brake drum;
- change-speed gear members;
- pump torque take-up factor of the hydraulic element;
- torque take-up factor of the hydromechanical stage;
- efficiency of the hydraulic element;
- efficiency of the hydromechanical stage.

LITERATURE

1. Dr. Terplán Zénó: Mechanizmusok. (Mechanisms.) Tankönyvkiadó. Bp. 1962.

2. Előadások a "Mechanizmusok" tárgyköréből. (Lectures on the field of "Mechanisms".) Miskolc, 1960. jun. 17. Nehézipari Müszaki Egyetem magyarnyelvü közleményei. VII. kötet Bp. 1961.

3. Poppinga, R.: Stirnrad-Planetengetriebe. Stuttgart, 1949. Franc. Verlagshandlung.

4. РудеикО, Н. ф.: Планетарные передачи. Москва, Машгиз.I94з

5. Артоболевский, И.И.: Теория механизмов и машин. М.I95I

6. Артоболевский, И.И.: Механизмы. М. 95 . АН СССР.

7. Артоболевский, И.И.: Синтез плоских механизмов.М.I959.физ.

8. Beyer, R.: Kinematische Getriebesynthese. Berlin, 1953. Springer.

9. Добровольский, В.В.: Теория механизмов. М.I95I. Машгиз.

10. Franke, R.: Vom Aufbau der Getriebe. Band I. Berlin, 1948. Benth-Vertrieb, - Band II. Düsseldorf, 1951, Deutscher Ingenieur Verl.

11. Кореняко, А.С.-Кренштеин,Л.И.: Теория механизмов и машин.

12. Kozsevnyikov, Sz.N.: A mechanizmusok és gépek elmélete. (Theory of mechanisms and machines.) Bp. 1952. Tankönyvkiadó.

13. Kraemer, O.: Getriebelehre. Berlin, 1954. Verlag Techn.

14. Krause, R.: Grundlagen des systematischen Getriebeaufbaus. Berlin, 1952. Verlag Technik.

15. Krause, R.: Getriebelehre. Berlin, 1954. Verlag Technik.

16. Левенсон,Л.В.: Теория механизмов и машин.М.I948.Машгиз.

17. Levenson, L. V.: Kinematik und Dynamik der Getriebe. Berlin, 1952. Verlag Technik.

18. Tuplin, W. A.: Grenzen der Planetengetriebe. (The Limitations of Epicyclic Gears) Machine Design 36 (1964) Nr. 3.

19. Zink, H.: Lastdruckausgleich, Laufruhe und Konstruktion moderner Planetengetriebe. "Konstruktion" 16 (1964) Heft 1, 3, 5.

20. Klein, H.: Die Planetenrad-Umlaufrädergetriebe. München, 1962. Carl Hauser.

21. Krause, R.: Kritische Untersuchung der Wirkungsgrad-Berechnung einfacher Planetengetriebe. "Maschinenmarkt", 1961. Nr. 90.

22. Nikolaus, H.: Graphische Darstellung der Drehzahl-, Momenten-, und Leistungverteilung in einfach rückkehrenden Planetengetrieben. "Maschinemarkt" 1961. Nr. 90, 1962. Nr. 12.

23. Town, H. C.: Analysis and design of planetary gearing. Engineering Materials and Design 6 (1963) Nr. 1.

24. Seeliger, K.: Betriebsverhalten der Planetengetriebe-Systeme. Die maschine. - Antriebs-Getriebe-Technik 17 (1963) Nr. 9.

25. Chabert, G.: Montage des satellites doubles dans les trains planétaires. Société d'études de l'industrie de l'engrenage SEIE 1963. Nr. 41.

26. Brown, W.: Compound epicyclic gearing. Engng. Materials a. Design 6 (1963) Nr. 6.

27. Arnandow, K.: Über einige Grundfragen des Lastausgleiches bei Planetengetrieben. Maschinenbautechnik. 12 (1963) Nr. 8.

28. Büttner, P.: Planetengetriebe, Konstruktion und Herstellung, V.I.K. Berichte Nr. 137. Essen: Verein. Ind. Kraftwirtsch. 1963.

29. Jarchow, F.: Leistungsverzweigte Getriebe - Stirnrad- Planetengetriebe mit parallel liegenden Axialkolbengetrieben. VDI.-Z. 106 (1964) Nr. 6.

30. Seeliger, K.: Das Leistungsverhalten von Getriebe-Kombinationen. VDI.-Z. 106. 1964. Nr. 6.

31. Niemann, G." Stirnrad-Planetengetriebe mit Einfach-Schragverzahnung und Lastausgleich. VDI.-Z. 106 (1964) Nr. 6.

32. Stöbzle, K.: Schnellaufende Planetengetriebe für schweren und stosshaften Betrieb. VDI.-Z. 106 (1964) Nr. 6.

33. Вощинин, А.И.: Гидравлические и пневматические устроиства на строителных и дорожных машинах. Машгиз. I954.

34. Клибов, Л.В.: Кинематика и динамика дифференциалных передач. Техн. отчет 55 НАМИ.

35. Крюков, А.Д.: Планетарные передачи в транспортных машинах. МАШГИЗ, I950, Москва.

36. Крюков, А.Д.: Методика тягового расчета транспортных машин с гидромеханической трансмиссией. Труды ЛПИ, Ленинград, I954, номер 2.

37. Крюков, А.Д.: К исследованию однопоточных и двухпоточных гидромеханических передач. Труды ЛПИ. I95I. Ио. I.

38. Крюков, А.Д.: Методика расчета редукторов и коробок передач транспортных машин. Ленинград, ЛПИ. I954.

39. Прокофьев, В.Н.: Синтез кинематических схем гидромеханических передач. Докт. дисс. I952.

40. Руденко, Н.Ф.: Планетарные передачи, их теория, расчет и проектирование. НКТИ СССР. I960.

41. Шабанов,К.Д.: Исследования схем двухпоточных передач. Канд.дисс. Москва,1950.

42. Poppinga,R.: Das Planetengetriebe im Kraftfahrzeug. Automobiltechn. Zeitung, 1951. Nr.9.

43. Röhm,F.: Die Leistungsverzweigung der Umlaufgetriebe und ihre Kennlinien. ATZ. 1953. Nr.7.

44. Прокофьев,В.Н.: Автомобильные гидропередачи.Машгиз,1948.

45. Лапидус,В.И.: О свойствах двухступенчатых гидротрансформаторов. Автомобильная промышленность. 1959. Nr.12.

46. Drucker, E.M.: Les transmissions automatiques. "Rev. techn. automob." 1963. 18. Nr.209.

47. Buick's New Dynaflow Transmission, Product Eng. 19.1948. Nr.2.

48. Smith,D.H.: Single-stage Fluid Transmitter. "Bus Coach" 25. Nr.229.

49. Winchele, F.D. - Route W.D. - Kelley O.K.: Chevrolet Turboglide Transmission. SAE Transactions, 1959.

50. Jeff S.D.: Cartwright B.W.: Chrysler Torque Flite Transmission. SAE. Transaction 1958.

51. Gaching W.: Das Voith - Diwabus - Getriebe. ATZ. Nr.3. 1953.

52. Diwabus - Getriebe für Stadtomnibusse. ATZ . Nr.11. 1953.

53. Flynn H.O.: Powermatic - a new automatic transmission. SAE reprint, Nr.672, g.I. 1956.

54. Bolygómüves sebességváltók hatásfokának kisérleti meghatározása. (Experimental determination of the efficiency of change-speed gears with epicyclic gears). VDI.-Z. 1960.28.

55. Wolkenstein, R.: Bolygómüvek egyenletes terheléselosztása. (Uniform load distribution of epicyclic gears).

56. Jurek,A.: Verzweigung der Eingagsleistung in Planetengetrieben mit Doppelantrieb. Acta Technika, Bp.29. (1960) 3-4.

57. Jurek,A.: The Description of the "Jupiter" Hydromechanical Continously Varipale Speed Transmission. Period. Polytechn. Budapest, 4. (1960) 2.

58. Сунина,В.Я.: Определение передаточных чисел и к.п.д. планетарных передач. Вестник Машиностроения 42./962/4.

59. Thüngen H.: Berechnung von Planetengetrieben. ATZ. 65 (1963) 3.

60. Kelley, O.K.: Tips on Transmission Planetary Gear Design. SAE Journal 67 (1959) 3.

61. Kovácsházi E.: Ujabb gépkocsihajtómüvek. (Recent motor car driving gears). Mérn.Továbbk.Int. 3800. Bp.1960.

62. Kovácsházi, E.: Gépkocsihajtómüvek felépitése és szerkezeti elemei. (Design and structural elements of motor car driving gears). Bp.1959. Mérn. Továbbk. Int. 3729.

63. Решетов.Л.Н.: Планетарные передачи. Машгиз,1948.

64. Вейц,В.Л.-Чирев,В.И.: Планетарные передачи тяжелых станков. Станки и инструмент.1958.9.

65. Wolkeinstein, R.: Zur Frage der konstruktiven Gestaltung von Planetengetrieben. VDI. 1957. 7.

66. Alleman F.: Korszerü hajtómüvek. (Up-to-date driving gears). Bp.1959. Mérn. Továbbk. Int.

67. Шабанов,К.Д.: Двухпоточные передачи тр. машин.Машгиз,1962.

68. Everden Harold I. F.: Variable rativ gear transmission (RollsRoyce Ltd). (English patent). 80. (2), Nr. 911049, 21.11.62.

69. Polomski J.B.: Change-speed transmission (Borg-Warner Corp) (English patent). 80 (2) Nr. 907695, 10.10.62.

70. Quistgaard E.V. - Kronogard S.O.: Utvaxlingsanordning, sarskilt för bilar (Ab Volvo). (Swedish patent). Nr.181784. 4.12.62.

71. Kelley O.K.: Transmission (General Motors Corp). (USA Nr. 3030823. 24.4.62.

72. Hause G.K.: Split torque transmission (General Motors Corp). (USA patent) Nr. 3039325. 19.6.62.

73. Boite de vitesses automatique, notamment pour véhicules automobiles (Automotive Products Co. Ltd). (French patent). Nr. 1287705. 5.02.62.

74. Wickmann A.C.: Epicyclic power transmission mechanisms. (English patent) Nr. 915396. 9.01.63.

75. Boite de vitesse á engrenages épicycloidaux á transmission continue de puissance, munie d'un embrayage centrifuge de départ, notamment pour véhicles automobiles (Fiat Soc.per Azzioni) (French patent). Nr. 1290081. 26.02.62.

76. Lindsay J.D.: Transmission (General Motors Corp). (USA patent). Nr.3049945. 21.08.62.

77. Dodge A.Y.: Transmission. (USA patent) Nr.3048057. 7.08.62.

78. Snyder K.E.: Transmission (General Motors Corp.) (USA patent). Nr.3057225. 9.10.62.

79. Mamo A.C. Transmission (Borg-Warner Corp). (USA patent). Nr. 3049027. 14.08.62.

80. Hause G.K. - Kelley O.K.: Multi-phase transmission (General Motors Corp). (USA patent). Nr. 3062074. 6.11.62.

81. Wayman R.W.: Rear mounted transmission (Borg-Warner Corp) (USA patent). Nr. 3063308. 13.11.62.

82. Terplán, Z.: - Apró, F: Az egyszerü bolygómüvek teljesitményviszonyai. (Performance conditions of simple epicyclic gears). Nehézipari Müszaki Egyetem Magyarnyelvü Közleményei X.1964.

83. Terplán, Z.: A legegyszerübb epiciklikus hajtómüvek analitikai vizsgálatának különböző módszerei. (Different methods of the analytical examination of the most simple epicyclic driving gears). Nehézipari Müszaki Egyetem Magyarnyelvü Közleményei X.1964. Miskolc.

84. Terplán, Z.: A fogaskerekes bolygómüvizsgálatok alapjai. (Bases of gear--wheel epicyclic gear examinations). Bányászati Kutató Intézet Közleményei. VII.(1962) 2.

85. Jurek, A.: A belső fogazás egyes kérdései és a bolygómüvek áttételének grafikus szerkesztése. (Some problemes of the inside gearing and the graphic design of the transmission of epicyclic gears). MTA.Müsz.Tud. Oszt.Osztályközleményei IV.2. Bp.1962.

86. ifj.Sályi, I.: Mechanizmusok analizise (Analysis of mechanisms). (Egyetemi jegyzet). Bp.1961.

87. Balogh, A.: Berechnung der Geschwindigkeit der Planetengetriebe auf einer neuen Grundlage. Maschinenbautechnik 3 (1954). 12.

88. Miller J.A.: Transmission (Borg-Warner Corp). (USA patent) Nr.2990727, 4.07.61.

89. Kelley O.K.: Transmission (General Motors Corp). (USA patent) Nr.2995955, 15.08.61.

90. Moore, V.C.: Transmission (General Motors Corp). (USA patent) Nr.2995956. 15.08.61.

91. Balogh, A.: Bolygókerék számitása különös tekintettel szerszámgépekre és gépjármüvekre. (Calculation of the epicyclic gear with special attention to machine tools and motor vehicles). Bp.1950.

92. Winchell F.J.-Kelley O.K.: Transmission (General Motors Corp). (USA patent). Nr.3010278. 28.11.61.

93. Burtnett E.R.: Transmission mechanism (Borg-Warner Corp) (USA patent) Nr.3000234, 19.09.61.

94. Lueger Lexikon der Technik - "Umlaufgetriebe" DVA. Stuttgart, 1960.

95. Förster H.J.: Föttingergetriebe in Leistungsverzweigungen. VDI - - Vorschungsheft 444.B.Band 20. 1954. Düsseldorf.

96. Lévai, Z.: Analytische Untersuchung elementarer Planetengetriebe. Acta Technika Hung. XLIX. 3-4. 1964.

97. Lévai, Z.: Analytische Untersuchung komplexer Planetengetriebe. Acta Technika Hung. LIII. 1-2. 1966.

98. Lévai, Z.: Analyse der Planetengetriebe. Az Építőipari és Közlekedési Müszaki Egyetem Tudományos Közleményei XI. 1964. № 6.

99. Lévai, Z.: Analyse der Hydromechanischer Getriebe. Az Építőipari és Közlekedési Müszaki Egyetem Tudományos Közleményei. XI. 1965. № 3-4.

100. Lévai, Z.: Hatfokozatu bolygómüves sebességváltó tervezése. (The designing of the change-speed gear of six stages. Communications of the motor vehicle professorate of the Technical University for the Building Industry and Transport). ÉKME Gépjármüvek tanszék közleményei. Kézirat. 1964.

101. Lévai, Z.: Bolygómüves sebességváltómü. (Change-speed gear with epicyclic gear). Hungarian patent. LE-450.

102. Jurek, A.: Automobilok. (Motor cars). Müszaki Könyvkiadó Bp. 1963.

103. Лапидус, В.И.: Гидпомеханические передачи автомобилей. 1961.

104. Леваи, З.: Анализ и выбор принципиальных схем гидромеханических передач. Канд. дисс. 1956. Ленинград.